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Two essays on reputation effects in economic models

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Two essays on reputation effects in economic models

by

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CHAPTER 1. REVIEW OF THE LITERATURE ON REPUTATION MODELS

1.1. Introduction

In this section we review and discuss the literature on repeated games where some players might be able to establish and/or maintain a reputation for choosing certain actions. The rough idea of most reputation models is that if the same stage game is played finitely or infinitely many times, and if a *player's* opponents have some prior belief (or initial reputation) that this *player* will be taking the same course of action every time the stage game is played then the *player* may try to preserve and/or to develop this reputation. There are numerous economic examples where an economic agent might be willing to commit to take the same action over a number of periods. For instance, the central bank may always implement its announced policy to convince traders that it will keep its promises in the future. As another example, a producer may choose a high quality of its product to convince potential buyers to choose its brand. The question is whether, given superior information of an economic agent, he will be able to effectively commit to his desired strategy.

1.2. The Chain-store Game

Many of the first formal game theoretic representations of the reputation models have originated from the industrial organization literature. Scherer (1980) indicates that a multimarket seller might employ sharp price cutting practices in response to entry in one market to try to scare off its potential rivals in other markets. Thus, an aggressive response to entry is used for demonstration purposes and may deter entry of potential entrants in other markets fearing that their entry will be met by the same type of response. Selten (1978) formalized this reasoning and called his model a chain-store game. We present a slightly modified version of Selten's game.

First, consider a single stage version of the chain-store game. The game has two players, the incumbent and the entrant. The entrant moves first and decides whether to enter the market monopolized by the incumbent. Thus, in period 1 the entrant chooses between two actions: to enter or to stay out of the market. If the entrant decides to enter, then the incumbent has to choose what type of pricing policy to pursue. The incumbent has two options: to accommodate (sharing the market "peacefully") or to fight (sharp price cutting). If the entrant decides to stay out, then the incumbent is not called upon to move and the game ends. The payoffs of the incumbent and the entrant in that case are $a > 0$ and 0, respectively.

If the entrant enters, the incumbent gets a payoff of 0 if it meets the entry by accommodation, and -1 if it fights. The entrant's payoff is -1 if it enters and is fought, and $0 < b < 1$ if it enters and the incumbent accommodates. The extensive and normal forms of the game are presented in Figures 1.1 and 1.2, respectively. If entrant enters then the incumbent chooses between 0 if it accommodates and -1 if it fights. Obviously, entry will be followed by accommodation. Anticipating this response, the entrant chooses between 0 if it stays out and b if it enters; hence, the entrant will enter. Thus, the unique subgame perfect equilibrium of this game is for the entrant to enter and for the incumbent to accommodate.

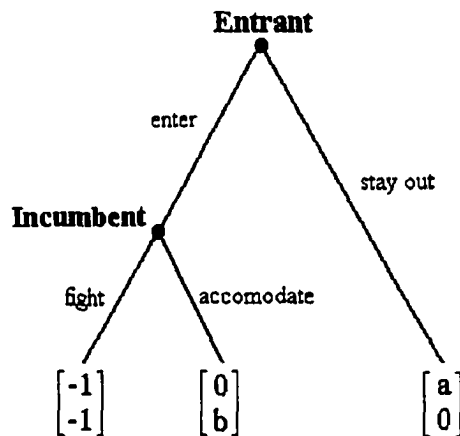


Figure 1.1. Single stage of the chain-store game.

		Entrant	
		<i>Enter</i>	<i>Stay out</i>
Incumbent	<i>Accommodate</i>	(0, b)	(a, 0)
	<i>Fight</i>	(-1, -1)	(a, 0)

Figure 1.2. Normal form of the single stage chain-store game.

This is not the only Nash equilibrium of the game, though. The strategy profile, where the entrant stays out and the incumbent fights if there is entry, is Nash equilibrium. It does not cost anything to the incumbent to fight entry since in the equilibrium entry never occurs, and the entrant stays out since entry is fought. But, the strategy of the incumbent to fight is an empty threat, since, if faced with entry, the incumbent's optimal choice is to accommodate.

Suppose the game just described is played by the single incumbent against N entrants sequentially. That is, the game of Figure 1.1 is played first against player N and the outcome becomes known to later entrants, then against player $N - 1$, etc. (Note, that we

index time backward.) The payoff to the incumbent is the discounted present value of the stage game payoffs (for some discount factor $0 < \delta < 1$), and the payoffs of entrants are those of a stage game. Consider the last stage of this game. Entrant 1 (the last entrant) chooses its action based on prediction of whether the entry will be met by fight or accommodation. Staying out is the best choice if entry will be met by rapacious response. But entrant 1 recognizes that if it enters then the incumbent has no reason to fight, since accommodation is the best response in the short run and the incumbent's choice in the last stage affects only the last stage payoff (no more opponents are left to demonstrate willingness to fight and hence no long run benefit from fighting). Thus, the last entrant enters no matter what the history of the game is up to the last stage and the incumbent shares the market peacefully.

Now, consider second-last market (entrant 2). If there is entry in this market the incumbent accommodates, since fighting is costly in the short run and does not affect the play in the last stage (by the logic of the previous paragraph). Anticipating that entry will be met by accommodation, the second-last entrant enters given any course of action taken by players in previous periods.

Carrying this argument backwards to the beginning of the game tree, Selten finds that

the unique subgame perfect equilibrium of the finitely repeated game is: the incumbent accommodates and all entrants enter for every possible history of the game.

Selten (1978) argued (and it seems to be a plausible argument) that if this game or a similar type of game were actually played, then we would expect quite a few potential entrants to stay out and for the chain store to fight some competitors which entered the market. He suspects that the incumbent would fight earlier entrants to discourage entry by later entrants. The earlier entrants, expecting this kind of response, would not enter. In the later stages of the game, he expects that entry would occur and would be met by accommodation. Since the expected behavior and the game theoretic predictions are in dissonance, he calls this situation the "chain-store paradox".

The presented arguments show that in the (subgame perfect) equilibrium of Selten's chain-store game the behavior of the incumbent and the entrants is not influenced by the outcomes in the previous markets. Kreps and Wilson (1982b) and Milgrom and Roberts (1982) have shown that the addition to the chain-store game of a realistic assumption of incomplete information on the entrants' part about the incumbent's payoff structure and/or the available actions may serve as a mechanism that will allow a connection of behavior in "otherwise independent markets". Kreps and Wilson (1982b) assume that the entrants may

entertain the conjecture that the incumbent may get some positive utility out of fighting. That is, they may put some initial weight on the event that the incumbent's payoff structure is such that fighting is a better short run response than accommodation (and not the payoff illustrated in Figure 1.1). Milgrom and Roberts (1982) consider a richer information structure. They assume that entrants believe that the incumbent's feasible set of actions may consist of a single response (fight or accommodate). That is, the entrants assign positive probability to the following three events: **i)** the incumbent has both actions (fight and accommodate) available after the entry occurs; **ii)** the incumbent has only action "fight"; **iii)** the incumbent has only action "accommodate". As the authors argue (Milgrom and Roberts (1982, p. 287), this captures the idea that "a predatory response in one period might be a part of a general aggressive pattern, and a cooperative response might be a part of a general cooperative pattern". In their model, the incumbent has another piece of private information: the short-run cost of fighting. To capture the fact that some entrants may enter regardless of the anticipated response by the incumbent, Milgrom and Roberts assume that the outside opportunity of each entrant (its utility from staying out of the market) is its private information and can assume such a value that entering is a strictly dominant strategy.

We try to briefly sketch a model which is a hybrid of the ones in Kreps and Wilson (1982b) and Milgrom and Roberts (1982). The following discussion is mostly drawn from Fudenberg and Kreps (1987) and Chapter 9 of Fudenberg and Tirole (1991).

As in the chain-store game, the game has $N + 1$ players. Player 0 is the long-run incumbent who has a monopoly in N distinct markets and faces potential entry in each of these markets by short-run firms (Players $N, \dots, 1$). The game has N stages. In the n -th stage, Player n , having observed the outcomes of all previous contests (stage games $N, \dots, n + 1$), chooses whether to enter or to stay out of the market n and the incumbent, in case entry occurs, decides whether to fight or to accommodate the entry. The outcome of this stage game becomes known to all subsequent players.

The entrants assess initial probability ρ , that the incumbent is "tough" meaning that its payoffs are such that it will fight entry in every market along the equilibrium path. The incumbent is "weak" with probability $(1 - \rho)$. The weak incumbent's payoffs for the stage game are as in Figure 1.1, for each possible outcome of the stage game. Each of the entrants can have two possible types, "tough" with probability q or "weak" with probability $(1 - q)$. Tough entrants always enter (this can be modeled by assuming that the strategy space of tough entrants is singleton, consisting of "enter"). The weak entrant's payoffs for

the possible outcomes of the stage game are as given in Figure 1.1. Each entrant's type is private information and is distributed independently of other entrants' types and of the incumbent's type. The incumbent's objective is maximization of the average discounted payoff which is a sum of discounted stage game payoffs.

Before characterizing the equilibrium, we discuss some of the properties that any equilibrium (we focus on a sequential equilibrium) of this game will possess. First, any equilibrium of this game will have the tough incumbent fight every entry along the equilibrium path and tough entrants enter for any history of the game. This is the case because the strategies indicated above are strictly dominant for the players. Second, each entrant's strategy has to be the short-run best response to the incumbent's strategy. Third, the weak incumbent accommodates in the last stage for every possible history of the game since there is no value of maintaining a reputation.

Note that the weak incumbent is willing to fight entry only if fighting will deter some of the future entrants and if the benefit from entry deterrence covers fighting costs. Thus, the weak incumbent fights entry only if the potential payoff from fighting $a(1 - q) - q$ exceeds the payoff from accommodating which is 0.

The unique sequential equilibrium outcome of this game has the following

characteristics:

i) Cost of maintaining a reputation exceeds its long-run potential benefit: $q > a/(a + 1)$

Weak incumbent accommodates to every entry and weak entrant enters if $p < b/(b + 1)$. Note, entry occurs early in the game and the weak incumbent's payoff tends to 0 as the number of stages increases;

ii) Maintaining a reputation has a potential long-run benefit: $q < a/(a + 1)$

In this case, there exists a number $n(\rho)$ such that the weak incumbent fights in the first $N - n(\rho)$ markets, and weak entrants in these markets stay out accordingly. The average payoff of the incumbent tends to $(1 - q)a - q$ as the number of stages increases.

Note, that in case ii) the long run opportunity cost of accommodation exceeds the short-run benefit of accommodation and this is what brings the reputation effect alive. Fudenberg and Kreps (1987, p. 545) emphasize that "it is the cost/benefit tradeoff, and not the frequency of play or the number of opponents per se, which is the key to understanding reputation effects".

Fudenberg and Kreps (1987) indicate that it may seem from the previous discussion that the weak incumbent favors the situation where the outcomes of its interaction in previous markets are made known to all future entrants. To show that this is not always the

case they compare payoffs to the weak incumbent for two different informational structures: informational linkage and informational isolation. The game is one of informational linkage if each entrant observes actions in all previous stages. If each entrant observes only actions in its own contest then this is a game of informational isolation. Note that it is very easy to solve the game with informational isolation since its equilibrium is just N copies of a single stage game, where weak incumbent always accommodates and the weak entrant enters if $p < b/(b + 1)$ (otherwise, it stays out).

If the incumbent has sufficiently high initial reputation ($p > b/(b + 1)$) then it will prefer the situation of informational isolation since in that case all weak entrants stay out and the weak incumbent nets $a(1 - q)$ from each market. Even if reputation is worth to invest in (case ii), the weak incumbent is worse off under the informational linkage since in that case the weak incumbent has to prove that reputation each period with probability of at least q which imposes a cost on the incumbent.

The informational linkage is preferred when $p < b/(b + 1)$ and $q < a/(a + 1)$. In this case, all types of entrants enter under the informational isolation while some of the weak entrants are deterred when markets are informationally linked.

1.3. Reputation with a Single Long-run Player

Fudenberg and Levine (1989) generalize and extend results obtained by Kreps and Wilson (1982) and Milgrom and Roberts (1982) to the class of games where a single long-run player faces a sequence of myopic (short-run) opponents who play only once and each of them observes all previous play. We introduce some definitions and notation before we discuss their findings. Consider a game where the long-run player one faces an infinite sequence of different short-lived player two's. Each period player one and that period's player two simultaneously select actions from finite sets A_1 and A_2 respectively. Each player's action is revealed at the end of the period. Let \tilde{A}_i denote the space of mixed stage game actions corresponding to the action space A_i . The unperturbed stage game is a map $g : A_1 \times A_2 \rightarrow R^2$, which gives player i 's stage game payoff g_i as a function of the realized actions. And let $g(\alpha) = g(\alpha_1, \alpha_2)$ denote the expected payoff corresponding to the mixed action α .

The *unperturbed repeated game* is the infinite sequential repetition of the unperturbed stage game played by the long-run player against different short-run opponents. In this game, the long-run player's objective function is the normalized discounted value of expected payoffs $(1 - \delta) \sum_{t=0}^{\infty} \delta^t g_1^t$, where δ is the long-run player's discount factor ($0 \leq \delta < 1$) and g_1^t is his stage game payoff in period t . Each period's player two

maximizes that period's payoff.

The history h^t in period t consists of the realized actions through period $t - 1$. Let H_t denote the set of possible histories of the game in period t . $H_t = (A_1 \times A_2)^t$. Since all the past realized actions are observed, each player can condition his play on the entire past history of the game. A mixed strategy for player one is a sequence of maps $\alpha_1^t : H_{t-1} \rightarrow \tilde{A}_1$, and a mixed strategy for period t player two is a map $\alpha_2^t : H_{t-1} \rightarrow \tilde{A}_2$.

Since period t short-run player cares only about that period's payoff, in any Nash equilibrium of this game each player two's choice of mixed strategy must be a best response to the anticipated strategy of the long-run player. That is, if we denote by $B : \tilde{A}_1 \rightrightarrows \tilde{A}_2$ the correspondence that maps mixed stage game actions of player one to the best responses of player two, each period's play must lie in the graph of B in any Nash equilibrium of the game.

Fudenberg-Kreps-Maskin (1988) show that a version of folk theorem holds for class of games with a single long-run player. Let

$$V_1 = \{v_1 = g_1(\alpha_1, \alpha_2) \mid (\alpha_1, \alpha_2) \in \text{graph}(B)\} \text{ and } v_1 = \max_{\alpha_1} \min_{\alpha_2} g_1(\alpha_1, \alpha_2)$$

where minimum in the definition of v_1 is taken over all strategies α_2 that are best response to some strategy of player one. That is, V_1 is the set of feasible payoffs for player

one consistent with the short-run player playing a best response to some pure strategy of the long-run player and v_1 is player one's minimax value in the game where player two is restricted to play a best response to some strategy of player one. Then, if player one is sufficiently patient (δ is close to 1) any payoff in V_1 that is larger than v_1 can be supported as a sequential equilibrium.

One of the objectives of the paper is to show that this kind of "folk theorem" prediction is not robust against slight perturbations of the informational structure of the game. In the *perturbed game* the long-run player has private information about his payoff function. Player one's payoff is identified with his "type" $\omega \in \Omega$, where Ω has countably many elements. Let μ denote a probability measure on Ω , which represents prior beliefs of short run players about long-run player's utility function. The set of actions available to each player in the stage game of the perturbed game, as well as the payoffs of the short-run player (corresponding to different action profiles), are the same as in the unperturbed game. But player one's period t payoff $g_1(\alpha_1, \alpha_2, \omega)$ may depend on his type. A mixed strategy for the long-run player in the perturbed game is a sequence of maps $\alpha_1^t : H_{t-1} \times \Omega \rightarrow \tilde{A}_1$. A mixed strategy for the short-run player is the same as in the unperturbed game.

Let ω_o denote the type of player one with payoff structure as in the unperturbed game. Type ω_o is sometimes called "normal" or "sane". The question of interest is whether this type can benefit from the short-run player's uncertainty about his payoff structure as compared to the case when the short-run player has complete information.

Stackelberg payoff of type ω_o of the long-run player is defined as $g_1^*(\omega_o) = \max_{a_1} \min_{a_2 \in B(a_1)} g_1(a_1, a_2, \omega_o)$. The *Stackelberg strategy* of player one is the strategy that attains this maximum.

Assume that for each action $a_1 \in A_1$, there is a type $\omega(a_1)$ for whom it is a strictly dominant strategy to play a_1 in each period of the perturbed game.

Define $g_1^*(\omega_o) = \max_{a_1} \min_{a_2 \in B(a_1)} g_1(a_1, a_2, \omega_o)$, which we call player one's *commitment payoff*.¹ The strategy that achieves this maximum a_1^* is called the *commitment strategy*. Type of player one for whom it is a strictly dominant strategy to play a_1^* in every period is called a *commitment type*. Denote $\omega^* = \omega(a_1^*)$.

If "normal" type of player one could publicly precommit to a pure strategy then he would choose strategy a_1^* . Then, the question is whether the long-run player can achieve his commitment payoff in the absence of the precommitment mechanism.

To achieve his commitment payoff the long-run player must convince the short-run

player to play the best response to a_1^* . And the short-run player bases his decision of choice of action on his assessment of likelihood of different actions of player one. The authors prove a lemma that describes the process of statistical inference by the short-run player if the long-run player chooses the same action in each period.

We introduce some notation and definitions before we restate this lemma. Each (possibly mixed) strategy profile (α_1', α_2') induces a probability distribution π over $(A_1 \times A_2)^\infty \times \Omega$. Let h^* denote the event that $a_t' = a_1^*$ for all t . Let π_t^* be the probability attached by short-run players to the event that the commitment strategy is being played in period t after a history h^{t-1} , i.e. $\pi_t^* = \pi(a_t' = a_1^* | h^{t-1})$. Note that π_t^* is a random variable since h^{t-1} is a random variable. For a given π , satisfying $0 \leq \pi \leq 1$, and for an infinite history h let $n(\pi_t^* \leq \pi)$ be the number of the random variables π_t^* for which $\pi_t^* \leq \pi$. Since an infinite history of the game is a random variable, so is n .

Lemma 1. Let $0 \leq \pi < 1$. Suppose $\mu(\omega^*) = \mu^* > 0$, and that (α_1', α_2') are such that $\pi(h^* | \omega^*) = 1$. Then

$$\pi \left[n(\pi_t^* \leq \pi) > \frac{\log \mu^*}{\log \pi} | h^* \right] = 0$$

and for any infinite history h such that the truncated histories h_t all have positive probability and such that a_1^* is always played, $\pi(\omega^* | h^t)$ is nondecreasing in t .

The lemma asserts that if there is a positive prior probability of the commitment type and if the commitment strategy is always played, then there is a fixed finite upper bound on the number of periods in which player two will believe it is unlikely to be played. Note, that the lemma does not assert that player two eventually becomes convinced that he faces commitment type of player one. The lemma shows that the short-run players become convinced that the long-run player will play as a commitment type. Fudenberg and Levine use this lemma to prove the main theorem of the paper which says that the long-run player's discounted average payoff in any Nash equilibrium will exceed the bound which converges to his commitment payoff as player one's discount factor converges to one. The theorem is robust against further perturbations of the game's information structure. That is if any other types of the long-run player are allowed, the assertion of the theorem holds. Also, the conclusions of the theorem hold if the game is repeated sufficiently many *finite* number of times.

The theorem is powerful since it gives a tight lower bound for the long-run player's payoff and is in contrast with the message of "folk theorem".

Note that the discussion above was devoted to a certain class of repeated games where each stage game is a simultaneous move game. The authors ask the question whether

reputation effects have power when the stage game is not simultaneous. They present an example of the game where long-run player does strictly worse than predicted by the results for the simultaneous move games.

Example: In the stage game, the short-run player moves first and chooses whether to buy or not to buy from the long-run player. If the short-run player chooses not to buy then the game ends and both players receive payoff of zero. If he buys, player one has to choose between high and low quality. If player one chooses high then both players receive payoff of one, while if player one chooses low, then player one's payoff is 3 and player two's payoff is -1. In this example player one would choose the strategy of always playing high if he could publicly commit to that strategy. The results for the simultaneous move stage game might suggest that in any Nash equilibrium of this game player one will obtain a payoff arbitrarily close to one. To prove that this intuition is false the authors find a sequential equilibrium of the game where along the equilibrium path the short-run players never buy and hence player one's payoff is zero.

In this example, a certain action (not buy) taken by the short-run player "hides" the action that the long-run player takes if purchase occurs. Hence, if the short-run player does not buy, the long-run player is deprived of the opportunity to play his commitment strategy

(producing high quality) and to convince the subsequent short-run players of his willingness to continue playing this action. Note that this problem is not present in the chain store game, since the action that "hides" the incumbent's strategy (stay out) corresponds to the commitment outcome.

Thus, for general stage games it is not true that the long-run player can almost ensure the payoff he could obtain if he was able to publicly commit to any strategy.

The authors offer two responses to this problem. The first is to examine perturbations of the game with the property that all information sets in the stage game are reached with positive probability. In this case, by playing commitment strategy in every period the long run player will eventually force the game to the commitment outcome.

The second response is weakening the main theorem for the simultaneous move stage game. The authors consider the class of games where each stage game is a finite extensive game of perfect recall without moves by Nature. To capture the fact that some strategies of player two might "hide" actions of player one at certain information sets, the authors introduce the notion of *observationally equivalent strategies*.

Definition: The set of strategies of player one $O(a_1, a_2)$ that are observationally equivalent to (a_1, a_2) is defined as $O(a_1, a_2) = \{a'_1 \in A_1 \mid (a'_1, a_2) \text{ leads to the same}$

terminal node as (a_1, a_2) .

In the example with choice of quality the strategies high and low are observationally equivalent given that player two does not purchase.

If the long-run player plays the same action a_1 in every period then player two will eventually believe that the long-run player will be choosing some strategy a'_1 from $O(a_1, a_2)$ and hence will play a best response to some strategy with support in $O(a_1, a_2)$.

This consideration leads to the following definition: for each a_1 , let $W(a_1) = \{a_2 \mid a_2 \in B(a'_1) \text{ for some } a'_1 \text{ with support in } O(a_1, a_2)\}$. That is, $W(a_1)$ denotes the set of player two's pure strategy best responses to beliefs about player one's strategy that are consistent with the information revealed when that response is played.

Let $g_1^* = \max_{a_1 \in A_1} \min_{a_2 \in W(a_1)} g_1(a_1, a_2, \omega_o)$. Strategy of player one that achieves this maximum is denoted by a_1^* . Type of player one for whom it is a strictly dominant strategy to play a_1^* in every period is denoted by ω^* . The authors show if there is a positive prior probability of type ω^* , then in any Nash equilibrium player one's average discounted payoff exceeds the bound that converges to g_1^* as δ tends to one.

Fudenberg and Levine (1989) extend their main theorem to the case when the stage game is n -player simultaneous move game. That is, in each period the long-run player

faces $n - 1$ short-run opponents. In any Nash equilibrium the strategies of $n - 1$ short-run players must form a Nash equilibrium in the game induced by fixing the strategy of the long-run player. The commitment payoff and action for player one can be defined with obvious modifications of the single short-run player case. The authors show that if there is a prior probability of the commitment type then in any Nash equilibrium the long-run player obtains at least as much as his commitment payoff for discount factors close to one.

The case when players have a continuum of strategies in each period is also considered in the paper. In this case, the basic simultaneous move model is unchanged with two exceptions. First, A_1 and A_2 are assumed to be compact metric spaces and the set of possible types of player one Ω is an arbitrary measure space. The payoff maps $g_1 : A_1 \times A_2 \times \Omega \rightarrow R$ and $g_2 : A_1 \times A_2 \rightarrow R$ are assumed to be continuous on $A_1 \times A_2$.

Working with a continuum of strategies and arbitrary measure space of types gives rise to two technical complications. First, it is not the case any more that if the short-run player assigns a large probability weight to the commitment strategy, he must play a best response to it. Second, it is not sensible that the short-run player puts a positive probability on the commitment strategy. Instead the authors assume that the short-run player places a positive probability on all neighborhoods of the commitment strategy. The authors consider

types ω_o belonging to a positive probability set Ω_o , such that each type ω_o has a different commitment payoff and commitment strategy:

The commitment strategy of player one $a_1^*(\omega_o)$ is the limit of the sequence a_1^m such that $\min_{a_2 \in B(a_1^m)} g_1(a_1^m, a_2, \omega_o) \rightarrow g_1^*(\omega_o)$.

The authors show that in any Nash equilibrium almost all types in Ω_o obtain more than a bound which converges to their respective commitment payoffs as discount factor approaches one.

In Fudenberg and Levine (1989) the analysis is concentrated on the case where there is a positive prior of the pure-strategy commitment type. In general, the long-run player can obtain a higher payoff if he could publicly precommit to a mixed strategy. For instance, in the version of the chain-store game discussed in Fudenberg and Kreps (1987) the strategy that the long-run player would most like to precommit is where he fights entry with minimum probability that deters weak entrants from entering the market.

Also, in many games of interest actions chosen by economic agents are observed with some noise. That is, the outcome of the stage game may not be a deterministic indicator of actions chosen. This situation might arise if the long-run player uses a mixed stage-game strategy, if there is a moral hazard, or if the stage game is a general extensive form game.

Fudenberg and Levine (1992) address these and other issues in their paper where they study reputation effects in repeated games where a single long-run player faces a sequence of short-run players but the long-run player's choice of stage-game strategy is imperfectly observed. This paper improves on Fudenberg and Levine (1989) in several respects. The authors find both lower and upper bound on the long-run player's payoff. If the stage-game has simultaneous moves, if commitment types for every mixed strategy belong to the support of probability measure over possible types, and if the game is generic then both the upper and the lower bound converge to the long-run player's Stackelberg payoff as the discount factor approaches one. This shows that reputation effects can have very strong implications for repeated games with imperfect information. The second improvement over their 1989 paper is that the authors consider the case where players' actions may be subject to moral hazard. Fudenberg and Levine (1992) show that imperfect observability does not change the basic reputation-effects intuition when the long-run player's actions are statistically identified. In particular, they prove that limits of upper and lower bounds are independent of amount of noise under statistical identification.

1.4. Reputation with Many Long-run Players

The focus of our discussion in the previous subsection was on the class of games where a single long-run or patient player faces a sequence of short-run opponents who are uncertain about long-run player's payoffs. That discussion showed that presence of this kind of incomplete information allows a patient player to obtain at least a commitment payoff in any Nash equilibrium of repeated game. A long-run player can achieve a payoff corresponding to the strategy that he would most like to commit (a commitment strategy) by mimicking behavior of the commitment type.

There are many economic situations, modeled as repeated games, where more than a single player is long-lived. For instance, the prisoners' dilemma game played by the same two players finitely or infinitely many times, two players involved in a bargaining process, etc. There are a number of differences that arise in the repeated game where a long-run player's opponent cares about future payoffs. As in the previous discussion we assume that player one is the player that might try to maintain a reputation for following some strategy and player two is his opponent. Recall that in the repeated game against myopic opponents the best possible commitment for the long-run player is the Stackelberg strategy, since the short-run opponent in each period plays the best response to that period's expected action. However, if player two is patient then he will not play the best response when he expects

that this course of action may lead to the punishment by player one. This drives a wedge between the highest payoff player one can obtain by publicly committing to a particular strategy (which, in general, will not be static) and the lowest payoff he receives in any Nash equilibrium of the perturbed game.

When player two is not myopic, the best player one can do by publicly committing to a strategy in general is not a static repetition of some action. In this case, player one might achieve more by committing to a history dependent strategy. For instance, consider repetition of the prisoners' dilemma game (Figure 1.3). Static Stackelberg strategy for this game is to defect with resulting payoff of zero. If player one faced a patient opponent and could publicly commit to a "tit-for-tat" strategy (play today the action that opponent played yesterday) then he would obviously be better off. Consider a case when there is a positive probability of a "tit-for-tat" type, that is, a type that starts with cooperation and in each period repeats the action chosen by opponent in the previous period. Suppose that normal type of player one (i.e., type of player one with payoffs as in Figure 1.3) tries to mimic a "tit-for-tat" type and starts with cooperation in period one. Suppose player two defects in some period t and continues defecting. Then normal type of player one will not have a chance to show that cooperative action of player two in some period will be followed by

cooperation in the next period. And hence, player one is unable to convince player two that he will be following a particular course of action for some contingency since it may never arise in equilibrium. The intuition of this example highlights the difficulty in maintaining a reputation for being a type that follows history dependent strategy. The problem comes from the *off-equilibrium* path behavior. Player one can convince his opponent that commitment strategy will be played *on* the equilibrium path. But it is impossible to demonstrate what would have been done along the *off-equilibrium* path.

	<i>Cooperate</i>	<i>Defect</i>
<i>Cooperate</i>	(2, 2)	(-1, 3)
<i>Defect</i>	(3, -1)	(0, 0)

Figure 1.3. The prisoners' dilemma.

The first paper that studies reputation models with two long-run opponents is Schmidt (1993). He considers a model similar to that of Fudenberg and Levine (1989). The only difference is that player one's opponent is a long-lived player who maximizes his discounted average payoff (with some discount factor δ_2). The author shows that in any

Nash equilibrium of the perturbed repeated game the normal type of player one can guarantee at least the payoff he would get from public precommitment to a static strategy that minimaxes player two, given that player one is sufficiently patient. This bound is good in some games (for example, the chain-store game), but does not impose any restrictions beyond those of individual rationality and feasibility in others (for example, the prisoners' dilemma). Schmidt identifies the class of games where this lower bound on equilibrium payoffs is the most player one could obtain by publicly committing to any strategy. These are *games of conflicting interests* in which the static strategy a player would most like to commit holds the opponent down to his minmax payoff. Note that in this class of games player one can not gain from the ability to commit to history dependent strategies since player two has to receive more than minmax payoff in any Nash equilibrium of the game. Schmidt only considers commitment types that play pure strategies and indicates that all results of the paper can be extended to mixed strategies by using methods developed in Fudenberg and Levine (1992). The author shows that if the condition of conflicting interests does not hold then there is a perturbation of the game and a sequential equilibrium of this game such that player one's payoffs are bounded away from his commitment payoff in the limit as δ_1 approaches one. That is, conflicting interests is a necessary condition for

player one to obtain as much from any equilibrium of the perturbed as he would get from the strategy he would most like to commit. Even if the game is not of conflicting interests, findings of the paper allow one to restrict the set of equilibrium outcomes as compared to prediction of the Folk Theorem. Schmidt provides an example illustrating this point, which is presented in Figure 1.4. This is not a game of conflicting interests since commitment strategy of player one is U while strategy that minmaxes player two is D . Folk theorem for this example predicts that any point in the vertically shaded area of Figure 1.5 can be sustained as an equilibrium of the repeated game. Suppose player one mimics type that always plays D . Then he will eventually convince opponent that D will be played in the future with high probability and player two will eventually play a best response against this action. If player one is sufficiently patient he can guarantee at least payoff of 2 in any Nash equilibrium of the perturbed game. Thus, results of the paper allow one to reduce the set of equilibrium outcomes to the double shaded area of Figure 1.5.

	L	M	R
U	(3, 2)	(0, 1)	(0, 1)
D	(0, -1)	(0, -1)	(2, 0)

Figure 1.4. Example of a game with non-conflicting interests.

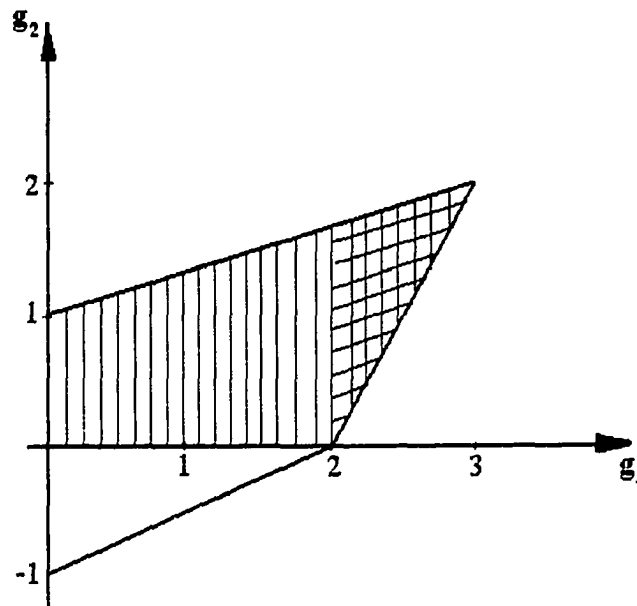


Figure 1.5. Vertically shaded area - the set of the Folk theorem predictions;

Double shaded area - equilibrium payoffs that survive Schmidt's criterion.

Schmidt shows that there is a fundamental difference between the games with a single long-run player facing a sequence of short-run opponents and games with many long-run players. To prove this point he presents a game where player one can be infinitely more patient than his opponent and shows that this game has an equilibrium which violates Fudenberg and Levine's (1989) lower bound for games with a single patient player.

Cripps, Schmidt and Thomas (1996) obtain a lower bound on equilibrium payoffs that improves on that of Schmidt (1993). Their model closely follows Schmidt (1993). To

discuss their results we will need some formal definitions.

Let $M(\hat{a}_1) = \{\alpha_2 \in \tilde{A}_2 \mid g_2(\hat{a}_1, \alpha_2) \geq \min \max g_2\}$ be the set of strategies of player two that yield at least as much as $\min \max g_2$ against \hat{a}_1 . And let

$a_1^* \in \arg \max_{a_1} \min_{\alpha_2 \in M(a_1)} g_2(a_1, \alpha_2)$. The corresponding commitment payoff is defined as

$g_1^* = \min_{\alpha_2 \in M(a_1^*)} g_1(a_1^*, \alpha_2)$. The main result of the paper states that if there is a positive

probability of commitment type ω^* of player one whose strictly dominant strategy is to play a_1^* in each period and if player one is sufficiently patient then he can guarantee almost g_1^* in any Nash equilibrium. The intuition of this result is the following. Normal type of player one can mimic type ω^* by playing a_1^* in each period. This will eventually convince player two that action a_1^* will be played in the future with high probability. If player does not play a best response to an expected action, that gets very close to a_1^* as time progresses, then player two receives less than his minmax payoff in the continuation game.

But this contradicts the fact that every player has to receive more than his minmax payoff in any Nash equilibrium of the game. Hence, player two will eventually play a best response against a_1^* and player one will be able to guarantee at least g_1^* if he is sufficiently patient. The authors also show that in any game with two possible types of player one, normal type and any type that follows a finitely complicated history dependent strategy

(finite automaton), there is a probability assignment over these types such that sufficiently patient player one gets an amount arbitrary close to g_1^* in some equilibrium of the perturbed game. Thus, g_1^* is tight in a sense that it is the highest lower bound on equilibrium payoffs that is robust against all possible informational perturbations.

Cripps, Schmidt and Thomas (1996) consider the battle of the sexes game (Figure 1.6) to illustrate the usefulness of their results to restrict the set of equilibrium outcomes. If player one plays U with probability $\frac{3}{4}$ and D with probability $\frac{1}{4}$ then player two is held down to his minmax payoff of $\frac{3}{4}$. If there is a positive probability of player one type who every period plays strategy that minmaxes player two, then in any Nash equilibrium of the game sufficiently patient player one can guarantee at least $2\frac{1}{4}$ (this set of equilibrium payoffs is depicted as the shaded area in Figure 1.7). Contrast this with prediction of the Folk Theorem that any outcome in the triangle ABC in Figure 1.7 can be supported as an equilibrium outcome of the repeated game.

	L	R
U	(3, 1)	(0, 0)
D	(0, 0)	(1, 3)

Figure 1.6. The battle of the sexes.

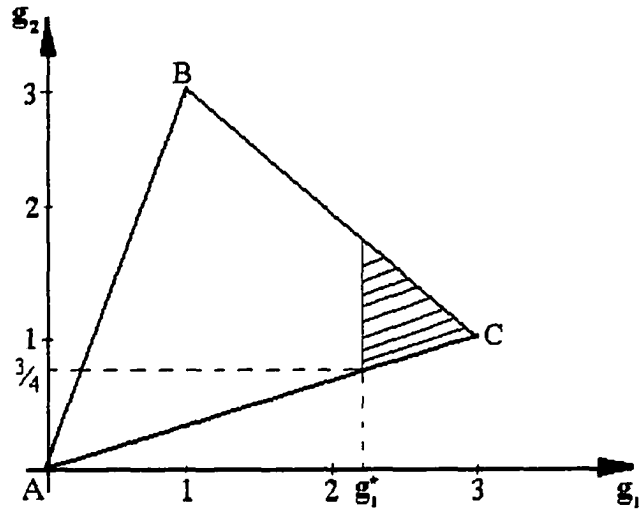


Figure 1.7. The set of equilibrium payoffs that survives Cripps et al. (1996) criterion.

The results discussed so far in this subsection require player one to be infinitely more patient than player two. This assumption may be justified if one is modeling a situation where a single player one faces a large sequence of long-run player two's, who observe all previous actions and alternate their play so that the first player two moves in periods $1, N + 1, \dots$, the second player two in periods $2, N + 2, \dots$, etc. In this situation player one plays in more future periods than any particular player two and hence cares more about future payoffs.

One justification for a player to become infinitely patient is that the lag between time periods gets smaller. If this interpretation of infinite patience is taken and if the same

physical players interact in each stage of the game, then it is the most plausible to assume that as one player becomes infinitely patient so does the other. The interesting question is whether reputation effects will still allow one to reduce the set of possible predictions if players are equally and infinitely patient. This issue is addressed in Cripps and Thomas (1995) who consider a general repeated simultaneous-move game between two long-run players that maximize Banach limits of average payoffs. As in other reputation models it is assumed that player two is uncertain about the type of player one and that player one may be a type who plays a particular pure or mixed action each period. Under the assumption on the presence of a particular player one type, the authors find a lower bound on equilibrium payoffs in any Nash equilibrium that is similar to the bound in Cripps, Schmidt and Thomas (1996). This bound is tight and is robust against further perturbations of the game.

In the beginning of our discussion in this subsection we have indicated that player one's best possible commitment when the opponent cares about future payoffs is in general not a static Stackelberg strategy. A history dependent strategy that punishes and rewards player two under different contingencies might yield a higher payoff to player one. A difficulty in establishing a reputation for playing such a strategy is that these rewards and

punishments are to be carried out under contingencies that may never arise on the equilibrium path. And player two cannot be convinced that his opponent is using this kind of strategy if rewards and punishments are not occasionally demonstrated. In the models discussed in this subsection player one could not improve his payoff by mimicking finitely complicated history dependent strategy. One common assumption of these models is the perfect observability of players' actions. Celentani et al. (1996) consider a general repeated simultaneous-move game with two long-run players where player two's stage-game action is imperfectly observed. The authors assume that at the end of each period a stochastic public outcome is observed by the players, but not the actions chosen by them in the stage game. The support of possible public outcomes is independent of the actions chosen by player two. Under this assumption contingencies occasionally arise that will allow player one to demonstrate rewards and punishments. Celentani et al. (1996) investigate whether player one can establish a reputation for playing a history dependent strategy in this environment. Below we present some definitions and assumptions of Celentani et al. (1996) to discuss their findings.

A behavior strategy has bounded recall if there exists a finite number N such that action in any period t depends entirely on the history between $t - N$ and $t - 1$. It is assumed

that there is a positive prior probability of each type of player one for whom it is a strictly dominant strategy to play some pure strategy of bounded recall.

The authors also assume that distribution over possible outcomes is statistically identified by the stage-game strategy of player one. If this assumption is not satisfied then player two can play a stage-game strategy that will prevent him from learning certain strategy of player one.

To ensure feasibility of rewards it is assumed that there is a pure strategy profile that yields player two a payoff greater than his minmax.

Celentani et al. (1996) show that a patient player one gets at least as much in any Nash equilibrium of the game as payoff from public precommitment in an arbitrary large finite truncation of the game. Note that the strategy that player one would most like to precommit in a finite truncation of the game does not have to be static.

The authors also show that as $\delta_2 \rightarrow 1$ a patient player one receives the maximum feasible payoff that gives player two at least his pure strategy minmax payoff.

The bound obtained in the paper is in general higher than that of Cripps, Schmidt and Thomas (1996) for the case of perfect observability. For example, in the prisoners' dilemma game of Figure 1.3 results of Cripps, Schmidt and Thomas (1996) give the same

prediction as the Folk Theorem: player one's payoff in any Nash equilibrium is bounded below by zero. While under incomplete observability and assumption that there is a positive prior probability of commitment types playing bounded recall strategies, a relatively more patient player one can achieve a payoff close to $2\frac{2}{3}$. Similar inspection of the battle of the sexes game allows us to conclude that patient player one guarantee almost a payoff of 3 in any Nash equilibrium of the perturbed game.

Aoyagi (1996) obtains results similar to those of Celentani et al. (1996). He considers a repeated game between two long-run players. Normal type of player one maximizes the time-average (note, that this reflects complete patience of player one):

$$v_1(\sigma_1(\omega_o), \sigma_2 \mid \omega_o) = \liminf_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^{\infty} E^{\sigma_1, \sigma_2} [g_1(a_1^t(\omega_o), a_2^t) \mid \omega_o],$$

where σ_1 and σ_2 denote mixed strategies of players one and two respectively, $\sigma_1(\omega_o)$ is the mixed strategy of the normal type of player one, and E^{σ_1, σ_2} is the expectation operator with respect to mixed strategies of players one and two. Player two maximizes normalized discounted average of stage payoffs (with some discount factor δ).

The sets of mixed strategies of a type of player one and of player two are denoted Σ_1 and Σ_2 respectively. Let $\bar{v}_1(\sigma_1, \delta) = \inf\{\bar{v}_1(\sigma_1, \sigma_2 \mid \omega_o) : \sigma_2 \in \Sigma_2 \text{ is a best response to } \sigma_1\}$, be the worst payoff of player one if he can publicly commit to strategy σ_1 . Player

one's *dynamic Stackelberg payoff* relative to $\Sigma'_1 \subset \Sigma_1$ is: $V_1(\Sigma'_1, \delta) = \sup_{\sigma_1 \in \Sigma_1} \bar{v}_1(\sigma_1, \delta)$. That

is, $V_1(\Sigma'_1, \delta)$ is player one's largest payoff if he could publicly commit to any strategy in Σ'_1 .

Let $\tilde{\Sigma}_1$ denote the set of *trigger strategies*. A strategy σ_1 belongs to $\tilde{\Sigma}_1$ if and only if there exists some finite number n , a sequence of pure actions of player two $\{a_2^1, \dots, a_2^n\}$ and a sequence of mixed actions of player one $\{\alpha_1^1, \dots, \alpha_1^n\}$ such that σ_1 cyclically plays $\{\alpha_1^1, \dots, \alpha_1^n\}$ in that order as long as player two plays $\{a_2^1, \dots, a_2^n\}$ in that order, and σ_1 prescribes minmaxing player two forever after any deviation of player two from the prescribed cyclical behavior.

Let Σ_1^\dagger denote the subset of strategies of player one defined similarly as strategies in $\tilde{\Sigma}_1$ with the only difference that strategy of player one prescribes to return to the original cycle after a deviation in the event that player two has played his cycle some finite number of times. That is, player one *forgives* his opponent if he observes this event. It is assumed that there is a positive prior probability of types for whom it is a strictly dominant strategy to play some strategy from Σ_1^\dagger . By mimicking one of these types player can guarantee at least a bound specified in the paper.

Aoyagi finds a lower bound on player one's payoffs in any *extensive-form trembling*

hand perfect equilibrium. For the sake of completeness, we present a definition of that equilibrium concept in Appendix I. We give a version used by Aoyagi, which is slightly different from the common definition.

Aoyagi shows that in any extensive-form trembling hand perfect equilibrium of the perturbed game player one's payoff is at least as high as $V_1(\Sigma_1^*, \delta)$ which is equal to $V_1(\tilde{\Sigma}_1, \delta)$. He also proves that as the discount factor of player 2 tends to one $V_1(\Sigma_1^*, \delta)$ approaches player one's maximum payoff in the individually rational set. Thus, if player two is sufficiently patient a completely patient player one can guarantee (in any extended form trembling hand perfect equilibrium) almost a payoff he would get from optimal public precommitment.

The reputation models that we have discussed up to this point in our review concerned only repeated games where a long-run player faces either a sequence of short-run opponents or a long-run opponent that can be equally or less patient. This class of games is a small subset of dynamic games. In general dynamic games some contingencies might arise that will effectively end the game for some players or the stage game played each period may not be static.

Celentani and Pesendorfer (1996) investigate whether reputation effects allow one to

narrow the set of predictions for a certain class of dynamic games. The authors consider a repeated game between one large player and a continuum of identical small players. Each period ($t = 1, 2, \dots$) all players randomize over finite sets of actions available to them (action sets of small players are identical). Each small player is characterized by his state, which is a private information of a small player except for the first period. The *public history* of the game at time t consists of the realizations of: i) actions chosen by a large player in the previous periods, ii) the probability measure over the Cartesian product of sets of possible states and actions for all previous periods.² The *private history* of the game at time t for each small player consists of realizations of his strategy and states in previous periods, and the state in period t . At the beginning of each period all players observe the public history and each small player observes his private history. All players maximize their discounted average payoffs. A pure strategy for the large player is a sequence of maps from public histories to the set of available actions. A pure strategy for the small player is a sequence of maps from the Cartesian product of public and his private histories to the available actions. Behavior strategies are defined in the obvious fashion. The game is *anonymous* in the sense that a deviation by a set of small players of measure zero does not affect the public history of the game. For each small player, probability

measure over possible states in the next period depends on the action of the large player and the aggregate action of small players in the current period as well as small player's current action and state. The authors allow for the possibility that the function describing the law of motion of the state variables may be stochastic. Note, that the game we have discussed so far can be thought of as an unperturbed game where large player's payoff function is known to all players. In the perturbed game there is a positive prior probability of the large player types that play restarting strategies: follow some strategy for a large finite number of periods T , in period $T + 1$ restart strategy, that is follow given the state in period $T + 1$, and so on. The authors prove that equilibrium payoff of a patient large player in any Nash equilibrium of the perturbed game is bounded below by the time average that the large player could obtain if he had an option of public precommitment in a large finite truncation of the game.

To show how reputation effects affect the set of equilibrium outcomes for dynamic games, Celentani and Pesendorfer consider three models: the time consistency of the optimal government policy, the price setting behavior of the durable good monopolist and Matsuyama's (1990) trade liberalization game. We give a brief sketch of the first two models.

Example 1. The large player is a government and the small players are the households in the economy. The government chooses whether to use a tax on capital income or a distortionary labor tax to raise a certain amount of revenue. Based on the expected tax rates, the households choose the amount of capital investment (it is assumed that choice of investment is discrete). Given that the government is at least as patient as the households, the level of capital investment is suboptimal in the unique Nash equilibrium outcome of the unperturbed game. In the perturbed game, the government's payoff is higher in any equilibrium than in the unperturbed game and the government can induce households to choose the optimal level of capital investment.

Example 2. The large player is the durable goods monopolist and the small players are buyers. There are two types of buyers (low and high valuation). The monopolist's cost of production is equal to zero. Each period the monopolist sets a price. If the buyer has not acquired the good in any of the previous periods he chooses whether to buy or not. If the buyer buys in some period then he moves to "state" where he stays forever and receives discounted value of the difference between the price paid and his valuation. If buyers are sufficiently patient, then the monopolist's payoff in any subgame perfect equilibrium of the unperturbed game is equal to the value of the good to the low valuation consumer. While in

the perturbed game the monopolist can guarantee a payoff very close to the total consumer surplus.

Celentani and Pesendorfer (1996) also investigate the case when the large and small players are equally patient. The lower bound on the equilibrium payoffs of the large player in any Nash equilibrium is the same as in the case when the large player is infinitely more patient than small players if the following *reversibility* condition holds. A function describing transition of the state variable satisfies reversibility condition if for every strategy of a small player and for every deviation from that strategy there exists a continuation strategy that allows to return to a path that is at least as good as the one corresponding to the original strategy. The reversibility condition is satisfied in the time-consistency model and the government's payoff is close to the best commitment in the large finite truncation of the game. This condition is violated in the durable good monopoly since once a buyer purchased a good he is locked in a permanent state. The authors find a sequential equilibrium of this game which yields the large player a payoff close to the value of the low valuation buyer.

CHAPTER 2. THE CHAIN-STORE GAME WITH MULTIPLE INCUMBENTS

2.1. Introduction

In the previous chapter we have presented a literature review on reputation models. As we have indicated, this literature originated from trying to resolve Selten's chain-store paradox. In the chain-store game a multi-market monopolist faces a finite sequence of potential entrants. Each entrant observes the actions taken in the previous markets and chooses whether to enter the market monopolized by the incumbent firm. If there is an entry the monopolist has two options: to fight the entry or to accommodate. Accommodation is the best short-run response for the incumbent, given the entrant entered the market, whereas fighting is an optimal policy only if it deters some of the future potential entrants. In the unique subgame perfect equilibrium of the complete information game all entrants enter and the incumbent accommodates, given any history of the game. Kreps and Wilson (1982b) and Milgrom and Roberts (1982b) have shown that addition of a small amount of incomplete information is sufficient to overcome this paradoxical game-theoretic prediction. In particular, they show that if entrants assign even a small probability to the event that the incumbent is a "tough" type who prefers a price war to accommodation, then the incumbent will fight in early markets to preserve a reputation for

"toughness". Since in later periods the value of having a reputation for being "tough" is low (because there are fewer future competitors), this reputation breaks down in one of the last stages of the game. The higher the initial reputation for "toughness" the more of early entrants are deterred and, hence, the higher the payoff to the incumbent.

One limitation of the chain-store game is that each of the markets with potential entry is monopolized by a single firm. While in some situations this might be a reasonable assumption, there are numerous examples when a market is comprised of multiple established firms (this may be due to the fact that monopolization is against the law). For instance, if we consider large retail stores and pick a random city in the USA, then it is very likely that one can find K-Mart in addition to a Wal-Mart store.

Our paper extends the basic chain-store game to the multiple-incumbent case. We consider a hypothetical situation where two established firms operate retail stores in a finite number of geographic locations (markets). These chain-stores sell products that are imperfect substitutes. Each of these markets is threatened by an entry of a different potential competitor. If the third store is opened then it will be selling an imperfect substitute with the products offered by the two chain-stores. Potential entrants are faced in a sequence and outcomes in previous markets become known to all later players.

Following entry, each of the two chain-stores simultaneously and independently decides whether to fight the entry (for instance, a sharp price cutting) or to accommodate (share market peacefully with the new entrant). If a potential competitor decides to stay out, then the two chain-stores split market profits. The entrant prefers to open a store only if neither of the chain-stores fights. If the entrant enters, then in the short-run each chain-store strictly prefers to accommodate no matter what the other chain-store's action is. Fighting by even a single chain-store imposes short-run losses on both of them. In Section 2.2 we give a detailed description of the game where potential competitors are completely informed of the payoff structures of the two incumbent firms (the complete information game), and show that the unique subgame perfect equilibrium is for both incumbents to accommodate and for all entrants to enter given any history of the game. That is, none of the potential competitors is deterred from entering.

We also investigate whether the same behavior emerges if each of the incumbents may have an initial reputation for being a "tough" type that will fight entry in each market along any equilibrium path. In the model of Section 2.3 each of the incumbents (as well as entrants) is uncertain about its counterpart's type. That is, each incumbent is uncertain whether the other incumbent enjoys fighting. As one would expect, the presence of initial

reputation deters entry under some parameterization of the model.

Our work is closely related to the entry deterrence literature.³ An extensive research in this area of industrial organization has started with the seminal contributions by Bain (1956), Modigliani (1958) and Sylos-Labini (1962). These early treatments of entry prevention were different variations of the limit-pricing model. The essential assumption of these models is that potential entrants believe that established firms will maintain the same price as the pre-entry level. Then, an incumbent or colluding incumbents may set such a low pre-entry price level that entry is discouraged. These models have received strong criticism. The incumbent firm, faced with the fact of entry, will find it in its own interest to raise the price above the pre-entry level. In other words a commitment of the incumbent firm to sustain the same price level is not credible.⁴ Spence (1977) and Dixit (1979, 1980) suggested that the threat to meet entry by aggressive policy can be made credible by pre-entry accumulation of sunk capital, so that the fulfillment of the threat is ex post (after the entry) optimal.

Other early examinations of entry prevention are Schmalensee (1978) and Eaton and Lipsey (1979). Both papers suggest that the incumbent or colluding incumbents may use product differentiation to preempt entry by filling up the product space.

Most of the early literature has concentrated on the case where a single incumbent or a group of colluding incumbents face a single entrant. However, many industries are characterized by more than a single established firm. Also, a number of firms enter industries during their evolution (Bernheim (1984)). A number of authors have investigated models with several incumbents and/or sequential entry. Our paper falls into this category but it differs from most treatments in sequential entry literature in that potential competitors enter different markets in our analysis.

Bernheim (1984) considers a highly stylized model of sequential entry where each incumbent has access to a wasteful and nondurable⁵ entry deterrence technology. The author shows that standard government policies aimed at decreasing industry concentration can have an opposite effect under some scenarios.

Gilbert and Vives (1986) analyze a model where an established oligopoly in a homogeneous good market faces a potential entrant that has to pay a fixed cost to enter. In the first stage of their game the incumbent firms simultaneously and independently choose their output levels. In the second stage the entrant decides whether to enter, and if it enters how much to produce. That is, the entrant acts as a Stackelberg follower in case of entry. Hence, entry is deterred if total output produced in the first stage exceeds a certain

threshold level. Entry prevention is a public good in this setting, since if one of the incumbents produces sufficient output to deter entry, the other incumbents benefit from entry deterrence. In our model entry deterrence is also a public good since any of the incumbents profits if its counterpart fights to preserve reputation for "toughness" and if that reputation deters entrants in later markets. Contributing to entry deterrence is costly in our analysis and, hence, each of the established firms wants to free ride on the entry preventing activities of the other incumbent. In Gilbert and Vives' model entry prevention is not necessarily costly. If equilibrium prescribes entry to be deterred, then each incumbent would like to contribute as much as possible to entry prevention (since additional output earns revenues). The authors show that underinvestment in entry deterrence never occurs in the equilibrium. Moreover, for some levels of entry fee (fixed cost) overinvestment may take place in a sense that the incumbents profits would be higher if they could collude and allow entry. This result is not very surprising since the incumbents would have a tendency to overproduce even in the absence of potential entrant.

Eaton and Ware (1987) consider an entry deterrence model with sequential capacity commitments where incumbent firms act noncooperatively. The authors find that the free rider problem does not arise in their analysis. McLean and Riordan (1989) analyze a model

similar in nature to Eaton and Ware's. In their model, firms enter sequentially and make one of the two irreversible technology choices in case of entry. One of the two technologies is the "detering" technology in the sense that a firm will choose it only to deter entry. The other technology is the "normal" technology; given the other firms' entry/technology decisions normal technology is always more profitable for a firm.⁶ After all firms have made their entry/technology decisions, firms that have entered share industry profits. Profit of each firm is a function of its technology choice, the number of firms that have chosen the normal technology and the number of firms that have chosen the deterring technology. McLean and Riordan find that under some scenarios collusion between early entrants to deter future rivals would be mutually beneficial, but does not occur in a non-cooperative equilibrium. That is, underinvestment in entry deterrence is a distinct possibility.

Waldman (1987, 1991) provides an intuitive explanation why there is no evidence of the free rider problem in Bernheim's, Gilbert and Vives' and Eaton and Ware's models, while in McLean and Riordan's analysis underinvestment can be the unique equilibrium. Waldman (1991) points out that a distinct feature of Bernheim's and Gilbert and Vives' analysis is that the total return to investing in entry deterrence occurs at a single point. That is, there is a single critical level of investment such that an epsilon increase in entry

deterrence activity induces the probability of entry to change from zero to one. Waldman (1987) reexamines Bernheim's and Gilbert and Vives' models by adding to them an uncertainty on the incumbents' behalf about the critical level of entry preventing technology (which eliminates the single critical level of investment). He finds that introduction of uncertainty into Bernheim's model causes the free rider problem to become an important factor in a sense that there is a strong tendency for the oligopoly to underinvest in entry deterrence, while addition of uncertainty to Gilbert and Vives' model makes overinvestment in entry prevention even more likely. In his 1991 paper, Waldman shows that removing the assumption of nondurability of entry deterrence investment from Bernheim's model may result in underinvestment.

Church and Ware (1996)⁷ consider a model of sequential entry into an industry with U-shaped average cost curve. They completely characterize the equilibrium market structure for linear demand and quadratic costs. In their analysis, a firm may deter entry of future rivals by producing "large" output. This is costly since the marginal cost is increasing. But it also confers a strategic advantage over future entrants, since a firm gets a larger share of the market by producing "large" output. Depending on which effect dominates, a firm will deter entry of future competitors or delegate that task to later

entrants.

In addition to the "first-mover" and absolute cost advantages, the incumbent firms may have an informational advantage over potential entrants. This asymmetry of information may result from the fact that the incumbents inhabited the industry for a long enough time to learn specific information on market demand and industry costs. One of the first treatments of informational asymmetry⁸ between incumbents and entrants is Milgrom and Roberts (1982a). They consider a model where the incumbent is better informed about its cost function than the potential entrant. In their model the incumbent charges a low pre-entry price to signal that its marginal cost is low, and hence the entrant will be in a strategically unfavorable position upon entry. Bagwell (1985) considers a model where the incumbent can use wasteful advertising to convey to the entrant that its cost is high. He finds that advertising is not used in the equilibrium. Thus, in these models of asymmetric information pre-entry action of an incumbent is a signal about its payoff function. This is also true of our model where an incumbent or incumbents may fight in early markets to signal to future competitors that fighting yields a higher payoff than accommodation.

Harrington (1987) extends and modifies the analysis of Milgrom and Roberts (1982a) for the case when the industry is initially comprised of multiple incumbents. In

Harrington's model, the incumbent firms are better informed about the industry-specific cost parameter than potential entrants. In the pre-entry period the incumbent firms simultaneously and independently choose their outputs. Potential entrants observe the first (pre-entry) period price. High price in the first period signals to potential entrants that this is a high cost industry and, hence, may discourage entry (in contrast to low price in Milgrom and Roberts' model). Based on their updated beliefs about the common (to both incumbents and entrants) cost function, the entrants make their entry decision in the second period. In case of entry, potential entrant incurs an entry fee and learns industry cost function. The third period is a quantity competition between incumbent firms and new entrants (if any). Harrington finds that the presence of asymmetric information allows noncolluding incumbents to deter entry that would have occurred under symmetric information. He also shows that an increase in the potential competition (number of prospective entrants) may result in a higher pre-entry price and welfare losses. The model in our paper and Harrington's analysis have a critical difference in the assumption on the information structure of the game. In Harrington's model the incumbents are perfectly informed about each other while in our paper each incumbent possesses information not known to all other players.

In our model we investigate whether both, one or none of the chain-stores would be willing to fight entry to preserve reputation for "toughness" and whether entry is successfully deterred. As we show in Section 2.3 this crucially depends on the absolute as well as relative initial reputations of the two incumbents. This reputation has value only if some of the future entrants are deterred. Note that it has value to both chain-stores. That is, even if only one of the chain-stores fights to maintain its reputation, the other chain-store may benefit from that reputation. And each chain-store prefers the other one to fight if that prevents entry in some of the future periods. One of the most interesting findings of the paper is that an increase in reputation for "toughness" can have a negative value. That is, over some range of initial reputations a weak incumbent's payoff is a decreasing function of its reputation. This is in sharp contrast with Kreps and Wilson's and Milgrom and Roberts' models where higher reputation deters more entrants and, hence, is always beneficial.

The rest of this chapter is organized as follows. In the next section we give a detailed description of the complete information version of our game. Section 2.3 contains an analysis of the finitely repeated incomplete information game. We give a complete characterization of the solution of the game with two periods and identify potential pitfalls

in trying to solve the game with more than two periods. We also find the lower bound on a sufficiently patient incumbent's payoff in the infinitely repeated version of the game. This bound is larger than the lowest payoff predicted by the Folk theorem. In Section 2.4 we conclude and outline some extensions and generalizations of our model. Some of the formal proofs are relegated to Appendix II.

2.2. The Complete Information Game

In our model we consider a hypothetical situation where two chain-stores have operations in a finite number N of markets (numbered from 1 to N), each with a different geographic location. These chain-stores, called incumbents S and T , sell products that are imperfect substitutes. In each of the N markets there is a single potential competitor that can open a third store, that will be selling an imperfect (but close) substitute with the products offered by the two chain-stores. The potential competitor in each location is a different firm. A competitor in location $i = 1, \dots, N$ is called entrant i . If an entrant decides to open a store, then each of the two chain-stores simultaneously and independently decides whether to fight or to accommodate the entry. Fighting may reflect a situation where an incumbent meets entry by sharp price cutting, heavy advertising campaign or any other response having such a negative effect on the entrant's profits that staying out would

be preferred to entry. An accommodating incumbent is willing to share the market peacefully with the new entrant. If a potential competitor decides to stay out, then the two chain-stores enjoy their respective shares of market profits. The extensive game form for the case when there is a single potential entrant is presented in Figure 2.1. The first element in the triplet at a terminal node is the payoff to incumbent S , second is the payoff to incumbent T and third is the entrant's payoff.

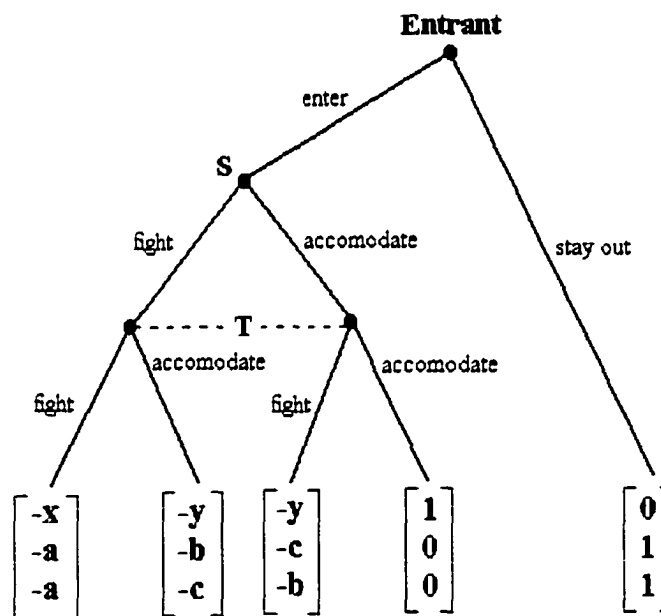


Figure 2.1. Single stage of the complete information game.

Now we give a detailed description of payoffs to the players in the game of Figure 2.1.

If the entrant chooses to stay out then this effectively ends the game and both incumbents receive a payoff of 1, while the entrant receives a payoff of 0. The entrant prefers to open a store only if neither of the chain-stores fights: $x > 0$ and $y > 0$. That is, if the entrant is certain that at least one of the incumbents will fight then it will stay out. The payoff of the entrant is lower in the case when both chain-stores fight as compared to the case when only one takes aggressive action: $x > y$. It is also assumed that $y > 1$ and $2y + 1 > x$. If at least one of the incumbents fights following entry then both incumbents receive a payoff lower than in the case when both accommodate: $a > 0$, $b > 0$ and $c > 0$. Thus, even in a situation when only one of the incumbents fights both are worse off compared with the case of peaceful response by both incumbents. If the entrant enters, then in the short-run each chain-store strictly prefers to accommodate no matter what the other chain-store's action is: $a > b > c$. That is, following entry, accommodation is a strictly dominant strategy for both incumbents. Thus, entry is followed by accommodation and, hence, yields a payoff of 1 to the entrant. While the entrant receives 0 if it stays out; and, hence, it will enter the market. Thus, in the unique subgame perfect equilibrium of the game with a single rival, the entrant enters and both incumbents accommodate.

Now suppose that the game of Figure 2.1 is played by the same pair of incumbents against a sequence of N different entrants. We index time backwards by assuming that incumbents S and T play first entrant N (period or stage N), then entrant $N - 1$ (period or stage $N - 1$), and so on. Each entrant observes the actions taken by the participants in all of the previous markets and makes its decision about the entry. The incumbents' total payoffs are the sums (undiscounted) of their respective payoffs in the N stages of the game. Application of backwards induction argument to this game yields a unique subgame perfect equilibrium: all entrants enter and both incumbents accommodate for every history of the game. The intuition behind this result is similar to the one of the perfect information chain-store game: each entrant i correctly foresees that the actions of the chain-stores in market i do not affect choices made in later markets (if any) and, hence, anticipates that both chain-stores will accommodate if entry occurs (since accommodation is a short-run best response). That is, the N markets populated by the two chain-stores are essentially independent.

2.3. The Incomplete Information Game

2.3.1. Description of the Game

In the previous section we have analyzed a repeated version of the game depicted in Figure 2.1. One would expect that at least one of the incumbents would fight in early markets to scare future entrants that their entry would be met by aggressive response. But we have shown that fighting never arises as a (subgame perfect) equilibrium behavior. As one suspects, this result is driven by the assumption of complete information. In this section we present a model where entrants initially assign some positive probability ρ_i to the event that incumbent $i = S, T$ prefers fighting to accommodation. That is, entrants are uncertain about incumbents' payoffs and allow for the possibility that entry will be met by aggressive response (possibly by both established firms).

Our formal model is a game with $N + 2$ players: incumbents S and T and N entrants. Incumbents S and T play the game form of Figure 2.1 against a sequence of N entrants. All later entrants are perfectly informed of the actions chosen in the previous stages of the game. The entrants assess some initial positive probability ρ_i that incumbent $i = S, T$ has a payoff structure such that fighting is a short-run best response for incumbent i . That is, it is assumed that incumbent $i = S, T$ can be one of two types: "tough" with probability ρ_i (with payoffs such that it will fight in every market along the equilibrium path) and "weak" with

probability $(1 - \rho_i)$ (with payoffs as in Figure 2.1). Each incumbent knows its payoffs but is uncertain about the payoffs to the other incumbent. We assume that incumbent $i = S, T$ and the entrants assess the same initial probability that incumbent $j \neq i$ prefers to fight in each stage of the game.

Each entrant can also be of two types. "Tough" entrants have a payoff structure such that they prefer to enter regardless of the incumbents' strategies and "weak" entrants have payoffs depicted in Figure 2.1. Each entrant is tough with probability q . We assume that all players' types are independent random variables. It is also assumed that $1 - q - b > a - c$.

We introduce some useful notation before solving for the equilibrium of the multi-stage game of imperfect information. Let p_i^k denote the probability assessed by the players in the beginning of stage k that incumbent $i = S, T$ is tough. For each $k = 1, \dots, N$, p_i^k is a function of the entire history of the game (actions taken by the players) up to and not including stage k . By our assumption, $p_i^N = \rho_i$.

The equilibrium concept that we use to solve our model is a slightly stronger version of Kreps and Wilson's (1982a) sequential equilibrium in that we impose additional restrictions on off-equilibrium beliefs. In particular, we require that the assessment be "sensible" in a sense that accommodation by any incumbent is seen as a sure sign of

weakness.

Before solving for our game, note that in any "sensible" sequential equilibrium tough entrants always enter and tough incumbents fight for any history of the game. Hence, in the following discussion we concentrate on the equilibrium strategies of weak players. For this reason, we frequently drop the adjective "weak" when it does not give rise to confusion.

2.3.2. One-period Game

In this subsection we consider the case when the two incumbent firms face a single entrant, that is the game form of Figure 2.1 is played only once. Note, that in any sequential equilibrium of this game both weak incumbents accommodate if entry occurs. There are no future potential competitors, and hence weak incumbents do not have any incentive to preserve reputation for "toughness" (since maintaining this reputation is costly in the short run). Thus, to solve for the sequential equilibrium we have to consider the weak entrant's choice between its two actions. The expected utility of the weak entrant if it enters is equal to

$$(1) Eu_1(\text{enter}) = p_S^1 p_T^1(-x) + p_S^1(1 - p_T^1)(-y) + p_T^1(1 - p_S^1)(-y) + (1 - p_S^1)(1 - p_T^1) \cdot 1.$$

And weak entrant nets a payoff of zero if it stays out:

$$(2) Eu_1(\text{stay out}) = 0.$$

Hence, the entrant strictly prefers to enter if and only if pair (p_S^1, p_T^1) lies inside the shaded area V of Figure 2.2 except for the prior probabilities that belong to the curve L connecting points $(0, \frac{1}{1+y})$ and $(\frac{1}{1+y}, 0)$. And weak entrant is indifferent between its two actions if (p_S^1, p_T^1) belongs to L . Thus, weak entrant enters if initial reputations of both incumbents are sufficiently low, and it stays out otherwise.

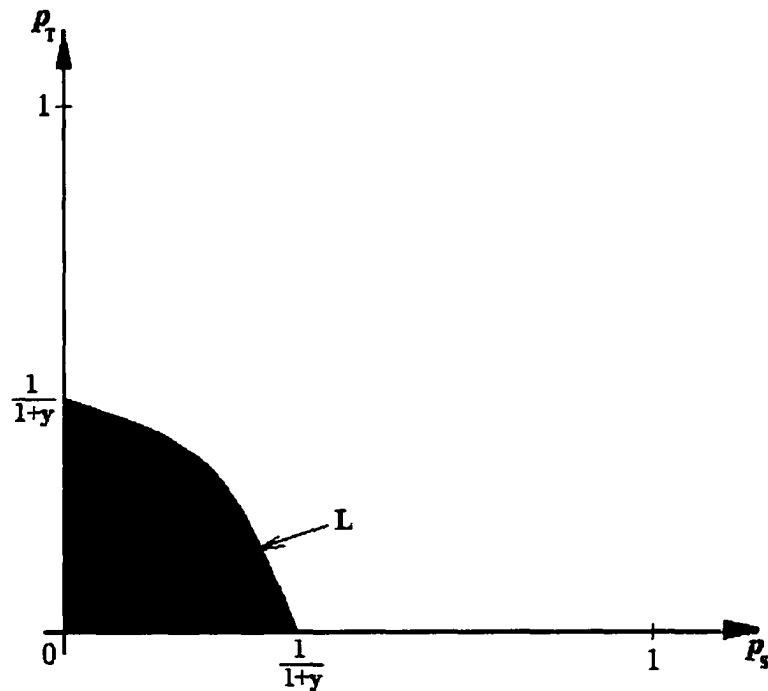


Figure 2.2. The set of initial reputations for which weak entrant enters.

2.3.3. Two-period Game

In the previous subsection we have shown that, when the two incumbents populate only one market, neither of the weak incumbents has an incentive to meet entry by aggressive response. This is the case since fighting is costly in the short-run and there is no return to preserving reputation for toughness. When two established firms face more than a single potential entrant, weak incumbents may benefit from pretending to be tough. Reputation of an incumbent is sustained until a later period only if that incumbent fought in all the previous markets. That is, a weak incumbent must incur a cost to maintain its reputation which has a value to that incumbent only if some of the future entrants are deterred. Note that if one of the weak incumbents chooses to fight in early markets to preserve its reputation and this deters some of the later entrants, then the other weak incumbent benefits from reputation maintenance of its counterpart. In other words, reputation of an incumbent can have a public good property in that provision of that reputation (its maintenance) benefits both weak incumbents. And since provision of reputation is achieved through costly fighting each of the weak incumbents has an incentive to free ride on the entry preventing activities of the other weak incumbent. Thus, in our model weak incumbents have both conflicting and common interests. Their common interest is deterrence of potential entrants, while the conflicting interest is that each of the

weak incumbents is reluctant to contribute to the "common interest".

To see which incentives of weak incumbents will dominate play of the game, we fully characterize equilibria of the sequential game with two periods. The equilibrium behavior of the players depends, among other things, on the initial reputations of weak incumbents. So, we present assessments that satisfy requirements of a "sensible" sequential equilibrium over a relevant range of prior probabilities (p_3^2, p_7^2) . The complete characterization of equilibrium assessments and the proofs that conditions on initial reputations are necessary and sufficient for the corresponding assessment to be a "sensible" sequential equilibrium are relegated to Appendix II. Note that for some ranges of priors there is a multiplicity of equilibria. Discussion of these ranges is also postponed to Appendix II.

Before presenting equilibrium assessments we list properties possessed by all equilibria. Recall that both weak incumbents accommodate in period 1 in any "sensible" sequential equilibrium. And tough incumbents fight in every market. Also, if there is no entry in period 2 then weak entrant 1 enters only if initial reputations lie inside area V and it stays out otherwise. Hence, when describing equilibrium strategy of weak entrant 1 we characterize its behavior at information sets following entry in period 2. If initial reputations lie outside area V then weak entrant 2 will stay out, otherwise its strategy will

depend on the actions of weak incumbents in period 2.

First, we consider a situation when *both* incumbents have a "sufficiently" high initial reputation: prior probabilities p_S^2 and p_T^2 are both strictly larger than $\frac{1-q-b}{a-c+1-q-b}$. The range of initial reputations satisfying these conditions is depicted as the shaded area in Figure 2.3. In this case the unique sequential equilibrium prescribes accommodation for both weak incumbents in period 2 (Equilibrium I in Appendix II). The intuition behind this equilibrium is the following. When *both* incumbents have a sufficiently high initial reputation for toughness then each weak incumbent is reluctant to fight for two reasons: i) incumbent $i = S, T$ has a high prior that the other incumbent is tough and, hence, that entry into market 2 will be fought with high probability. And this will be seen as a sure indication of toughness by entrant 1 given the strategy of weak incumbent $j \neq i$. Hence, incumbent i places a high probability on the event that weak entrant 1 will be deterred; ii) if weak incumbent i fights then it incurs additional cost $(a - c)$ in the event when incumbent $j \neq i$ is tough, which has a high prior probability. Thus, in the event when both incumbents are weak and entrant 2 is tough, which occurs with probability equal to $(1 - p_S^2) \cdot (1 - p_T^2) \cdot q$, entry of weak entrant 1 is not deterred and ex post (after the entry was accommodated) weak incumbents regret that none of them has fought.

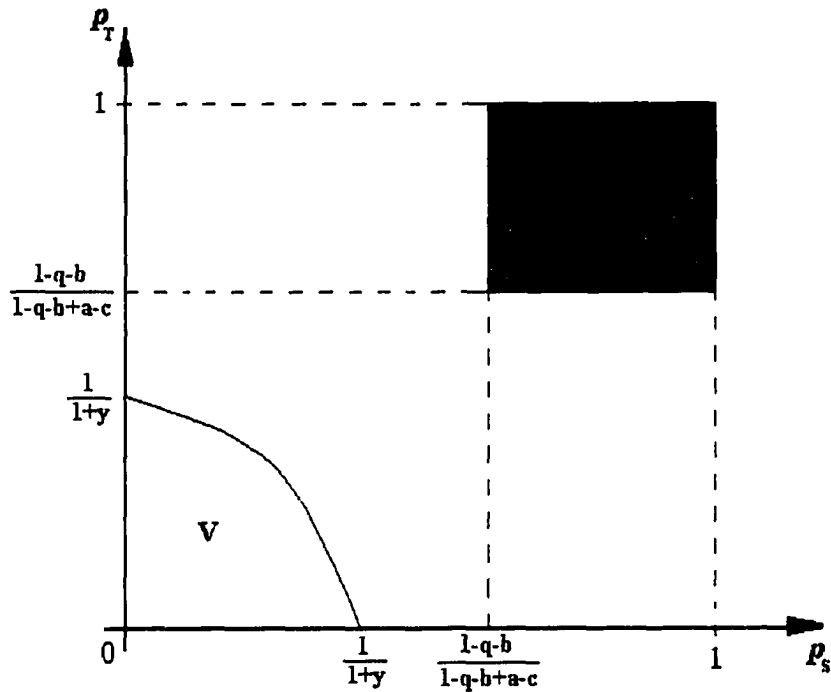


Figure 2.3. Both weak incumbents accommodate in period 2.

When initial reputations belong to the shaded area of Figure 2.4 both weak incumbents fight in a "sensible" sequential equilibrium (Equilibrium II in Appendix II). The intuition behind this equilibrium is the following. If only one weak incumbent fights in period 2 then this does not deter weak entrant in market 1, since both p_S^2 and p_T^2 are less than $\frac{1}{1+y}$. While if both weak incumbents meet entry in market 2 by aggressive response, then weak entrant 1 will stay out (since $(p_S^1, p_T^1) = (p_S^2, p_T^2)$ lies outside area V in this case). Since potential benefit of fighting outweighs its costs, both incumbents have an incentive to fight.

Roughly speaking, union of weak incumbents' reputations is sufficient to scare the future weak entrant while individual reputations are not.

When one of the incumbents has a "sufficiently" high initial reputation (strictly larger than $\frac{1}{1+y}$) and the other a "sufficiently" low one (strictly smaller than $\frac{1-q-b}{a-c+1-q-b}$), then in a "sensible" sequential equilibrium the incumbent with high reputation fights and the other incumbent accommodates.

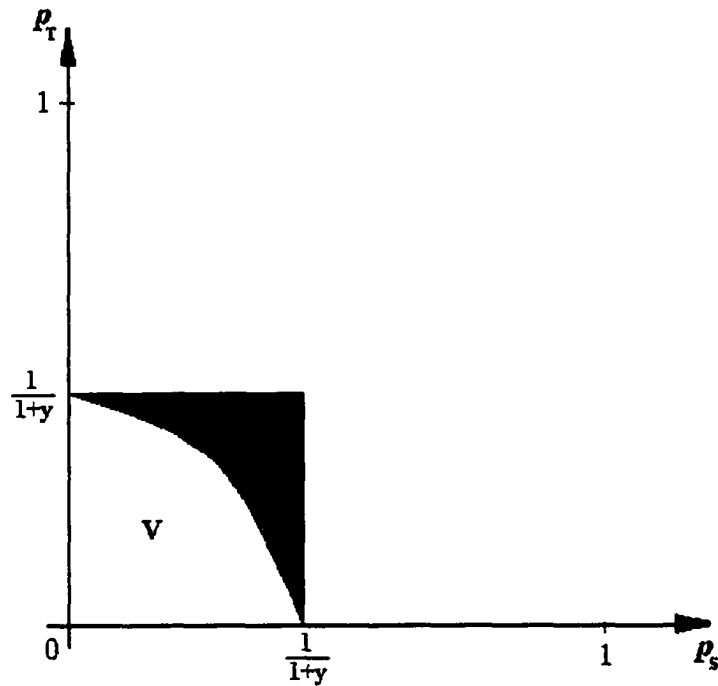


Figure 2.4. Both weak incumbents fight in period 2.

In this discussion, we consider only the case when incumbent T is the one with high initial reputation (Equilibrium III): $p_T^2 > \frac{1}{1+y}$ and $p_S^2 < \frac{1-q-b}{a-c+1-q-b}$ (shaded area in Figure 2.5). The case when incumbent S has a high reputation is completely symmetric in the sense that description of equilibrium becomes the same as that of Equilibrium III by switching subscripts S and T . In equilibrium III weak incumbent S free rides on the reputation of incumbent T that fights in period 2 irrespective of its type. Maintaining a reputation is a worthy investment for weak incumbent T for two reasons. First, it necessarily deters entry of weak entrant 1. And second, incumbent S is tough with relatively "small" probability and, hence, there is a small chance that entry will be deterred by actions of incumbent S . It is clear that weak incumbent S does not have any incentive to deviate from its strategy since incumbent T 's actions alone are sufficient to deter entry in the last period.

So far we have presented equilibria that have both incumbents play pure actions in period 2. There also exist equilibria where one or both of the weak incumbents mix over their actions in period 2. We divide our discussion of these equilibria into two parts. First, we present equilibria that correspond to ranges of initial reputations outside area V of Figure 2.2. Then, we discuss equilibrium behavior for initial reputations inside area V .

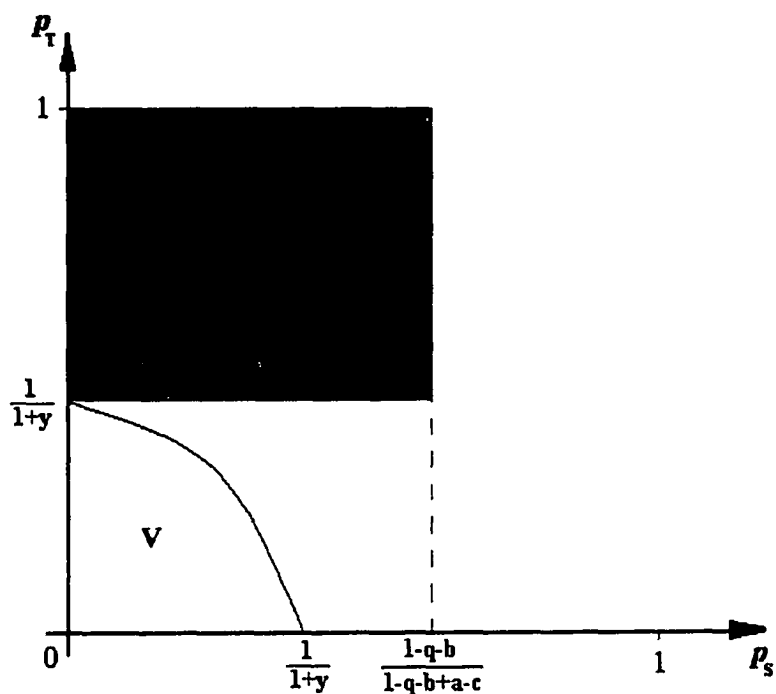


Figure 2.5. Weak incumbent S accommodates and weak incumbent T fights.

For prior probabilities (p_S^2, p_T^2) that belong to the shaded area of Figure 2.6, there is a "sensible" sequential equilibrium where in period 2 weak incumbent S mixes between its two actions and weak incumbent T accommodates with probability one (Equilibrium V in Appendix II). Note that if initial reputations belong to the range symmetric (along the 45° line) to the shaded area of Figure 2.6, then there exists an equilibrium symmetric to Equilibrium V. In Equilibrium V weak entrant 1 stays out if incumbent T fought in period 2 and mixes between "enter" and "stay out" if entry was fought by incumbent S and

accommodated by incumbent T . For weak entrant I to be willing to mix, the probability of fighting by weak incumbent S has to be chosen appropriately. In turn, randomization by weak entrant I is required to ensure indifference of weak incumbent S between "fight" and "accommodate".

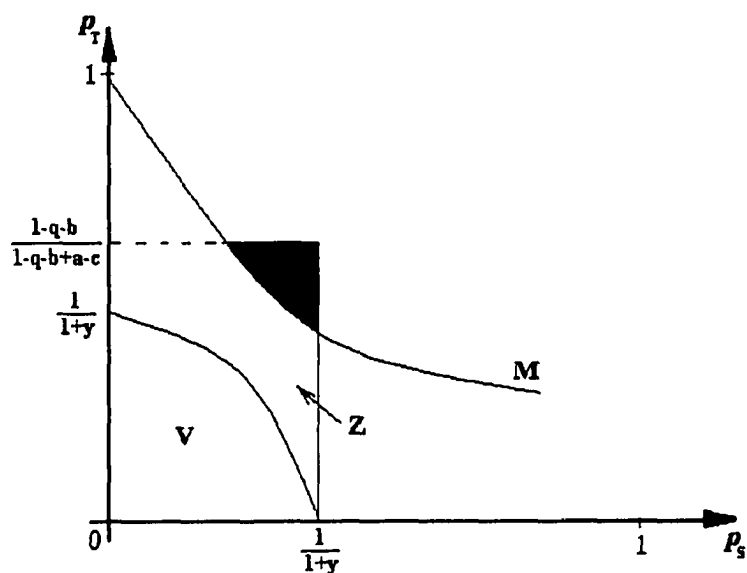


Figure 2.6. Weak incumbent S mixes and weak incumbent T accommodates.

If prior probabilities (p_S^2, p_T^2) belong to the shaded area of Figure 2.7

$$\left(\frac{1-q-b}{(a-c+1-q-b)(1+y)} \leq p_S^2, p_T^2 \leq \frac{1-q-b}{a-c+1-q-b} \right),$$

mixing by both weak incumbents in period 2 is a part of a "sensible" sequential equilibrium (Equilibrium VII in Appendix II). Randomizations in this equilibrium are chosen to ensure indifference of

weak incumbents between their two actions in period 2. Weak entrant 1's strategy in this equilibrium prescribes to stay out if at least one of the incumbents fought entry in period 2.

Now we consider a situation when initial reputations belong to area V of Figure 2.2. In this case it cannot be an equilibrium for both weak incumbents to fight in market 2. Suppose that equilibrium strategy prescribes for both weak incumbents to fight with probability one. Given these strategies, Bayesian updating yields posterior beliefs equal to the priors. Thus, the posterior beliefs lie inside area V and, hence, entry of weak entrant 1 is not deterred. Since fighting is costly in the short-run, both weak incumbents have an incentive to accommodate given weak entrant 1's strategy.

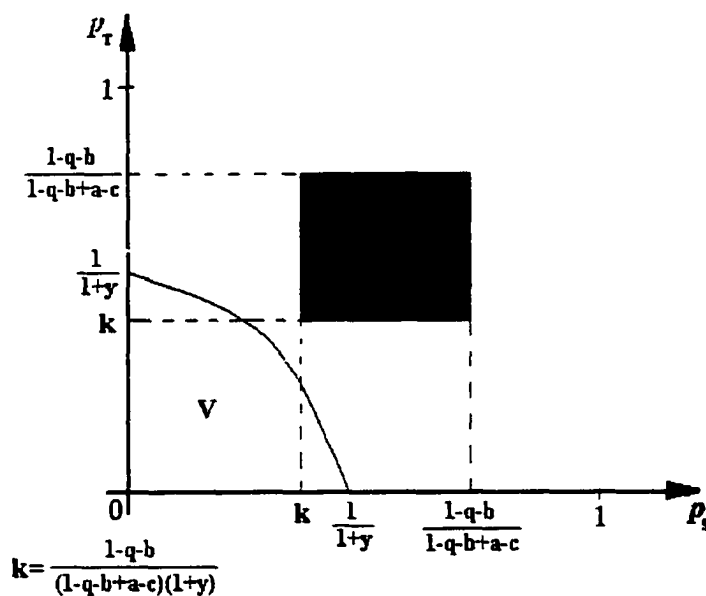


Figure 2.7. Both weak incumbents mix.

Neither can it be an equilibrium for both weak incumbents to accommodate with probability one in period 2. For if weak incumbents were using these strategies then fighting in period 2 is perceived as a sure indication of "toughness" and, hence, deters weak entrant 1. We will show that weak incumbent S (as well as weak incumbent T) can gain by deviating from its strategy and choosing to fight in period 2. Weak incumbent S , contemplating this deviation, recognizes that fighting will deter entry of weak entrant 1 with probability one, while if weak incumbent S were to accommodate then it would expect entry to be deterred with probability p_T^2 (incumbent T 's initial reputation). Thus, the gain from deviation due to entry deterrence is equal to $(1 - p_T^2)(1 - q)$. On the other hand, fighting imposes additional costs on incumbent S . Relative to accommodation, the increase in fighting costs is equal to $p_T^2(a - c) + (1 - p_T^2)b$. Since initial reputation of incumbent T is relatively small the gain from deviation outweighs the additional fighting costs and, hence, weak incumbent S 's strategy of accommodation in period 2 is not optimal.

It is also easy to show that it is not an equilibrium for one of the weak incumbents to fight in period 2 and for the other one to accommodate. Without loss of generality suppose that weak incumbent S fights and weak incumbent T accommodates in period 2. Given these strategies, weak entrant 1 stays out only if it observes that incumbent T fought in

period 2 (reputation of incumbent S is not sufficient to deter entry in period 1). That is, weak entrant 1's strategy does not depend on the action of incumbent S in market 2. And, hence, weak incumbent S would prefer to accommodate in period 2.

Thus, we have shown that if initial reputations belong to area V then it cannot be an equilibrium for both weak incumbents to play pure actions in period 2. That is, at least one of them should mix between its two actions. And a weak incumbent has to be indifferent between "fight" and "accommodate" to be willing to mix between these two actions. This, in turn, requires that weak entrant 1 mixes at some of its four information sets.

There are four possibilities for qualitatively different equilibria (Equilibria VIII, IX, X and XI in Appendix II) when initial reputations belong to area V . Which one of these equilibria ensues depends on initial reputations of the two incumbents. In Appendix II we provide a necessary conditions on prior probabilities for each particular assessment to be an equilibrium (note that over some ranges of initial reputations there is a multiplicity of equilibria). All four equilibria have a common feature in that weak entrant 1 enters if there was no entry in period 2 since the posteriors are equal to the priors in this case and, hence, lie inside area V .

Equilibria VIII and IX are symmetric (in the sense defined in the beginning of this

subsection). In these equilibria one of the weak incumbents mixes and the other fights with probability one in period 2. The range of initial reputations for which equilibrium VIII is sequential is depicted in Figure 2.8. Weak entrant 1 enters if at least one of the incumbents accommodated in period 2, and mixes between "enter" and "stay out" if entry was fought by both incumbents. Since one of the weak incumbents fights with probability one in period 2 and tough incumbents always fight, weak entrant 2 is certain that one of the incumbents will fight irrespective of its type and, hence, does not enter.

In equilibrium X both weak incumbents mix between their actions in period 2. Weak entrant 1 enters if at least one of the incumbents accommodated in period 2, and mixes if both established firms fought.

Equilibrium XI is similar to equilibrium X in that both weak incumbents mix in period 2 (the range of initial reputations is depicted in Figure 2.9). The qualitative difference between the two is that in equilibrium XI weak entrant 1 stays out at the information set following fight by both incumbents, enters if both incumbents accommodated and mixes if one incumbent fought and the other accommodated. Weak entrant 2's strategy in Equilibria X and XI depends on initial reputations of weak incumbents. In Equilibrium XI it enters only if initial reputations belong to the shaded area of Figure 2.10.

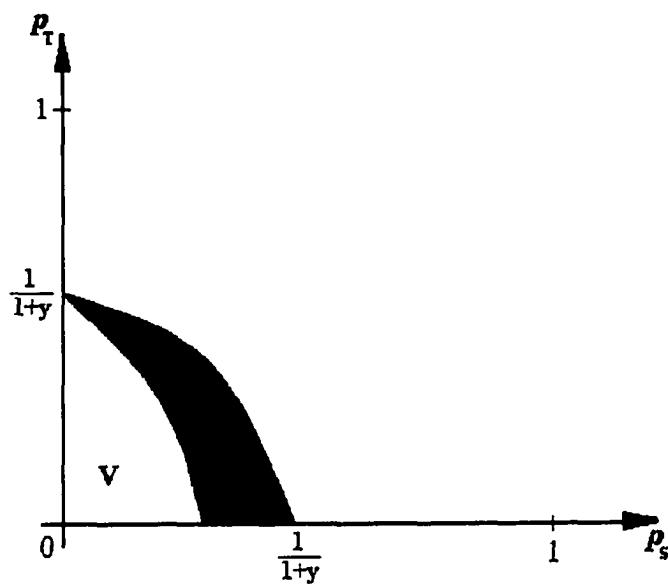


Figure 2.8. Weak incumbent S mixes and weak incumbent T fights.

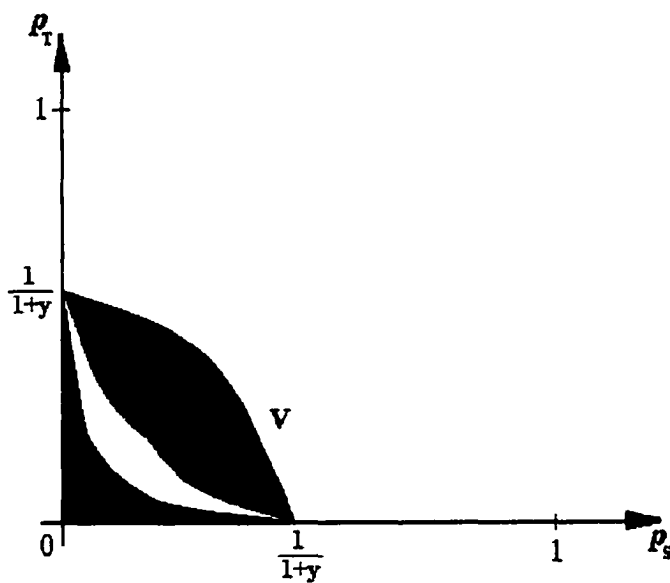


Figure 2.9. Both weak incumbents mix in period 2;

Weak entrant 1 mixes if exactly one incumbent fought in period 2.

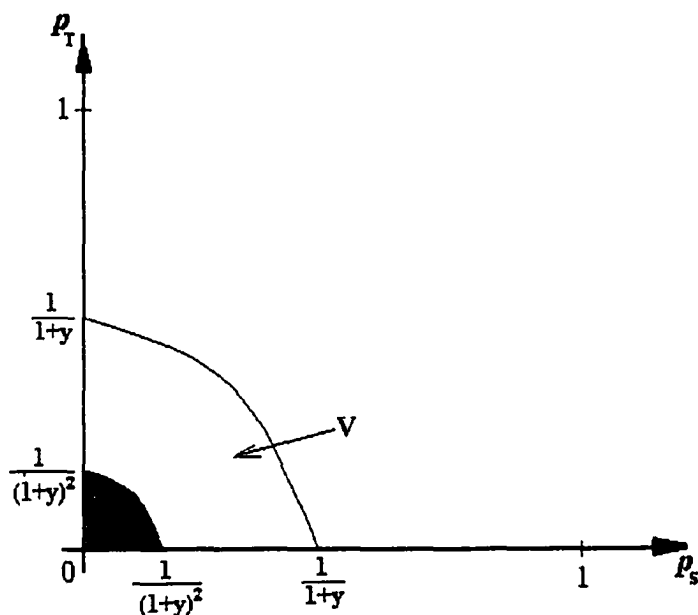


Figure 2.10. Weak entrant 2 enters if (p_s^2, p_T^2) belongs to the shaded region.

2.3.4. Value of Reputation in a Two-Period Game

In the incomplete information chain-store game the weak incumbent always benefits from having a high reputation for toughness. This is the case because a high reputation deters more entrants. Our next proposition shows that this is not always true when there is more than a single player that may try to establish and/or maintain a reputation.

Proposition: a) If initial reputation of incumbent i satisfies $p_i^2 > \frac{1}{1+y}$, then weak incumbent j 's ($j \neq i$) expected equilibrium payoff is a globally decreasing function of its reputation;

b) If initial reputation of incumbent i satisfies $p_i^2 > \frac{1-q-b}{a-c+1-q-b}$, then weak incumbent j 's ($j \neq i$) expected equilibrium payoff is a monotonically decreasing function of its reputation.

Proof of this proposition is a direct corollary of our derivations of equilibrium payoffs in Appendix II. In Figures 2.11 and 2.12 we graphed equilibrium payoff of weak incumbent S as a function of its reputation.

Figure 2.11 reflects a situation when incumbent T 's initial reputation satisfies $p_T^2 > \frac{1-q-b}{a-c+1-q-b}$. In this case the unique sequential equilibrium for "sufficiently" low values of incumbent S 's initial reputation ($p_S^2 < \frac{1-q-b}{a-c+1-q-b}$) prescribes fighting for weak incumbent T and accommodation for weak incumbent S in period 2 (Equilibrium III in Appendix II). And if initial reputation $p_S^2 > \frac{1-q-b}{a-c+1-q-b}$ then both weak incumbents accommodate (Equilibrium I in the Appendix II) in the unique sequential equilibrium. For a fixed initial reputation of incumbent T , weak incumbent S 's payoff in Equilibrium I is strictly smaller than in Equilibrium III (see Appendix II for equilibrium payoff expressions). Thus, over this range of incumbent T 's initial reputation weak incumbent S 's equilibrium payoff is a monotonically (and, hence, globally) decreasing

function of its reputation.

In Figure 2.12 we depict weak incumbent S 's equilibrium payoff as a function of its reputation for the case when $p_T^2 \in (\frac{1}{1+y}, \frac{1-q-b}{a-c+1-q-b})$. Note that under this condition there might be a multiplicity of equilibria for some values of initial reputation of incumbent S . In particular, when $p_S^2 \in (\frac{1-q-b}{(1+y)(a-c+1-q-b)}, \frac{1-q-b}{a-c+1-q-b})$ there are at least two "sensible" sequential equilibria (Equilibria III and VII in the Appendix II). Observation of payoffs for different equilibrium assessments indicates that the maximum equilibrium payoff of weak incumbent S is a decreasing function of its reputation, i.e. equilibrium payoff is a globally decreasing of reputation.

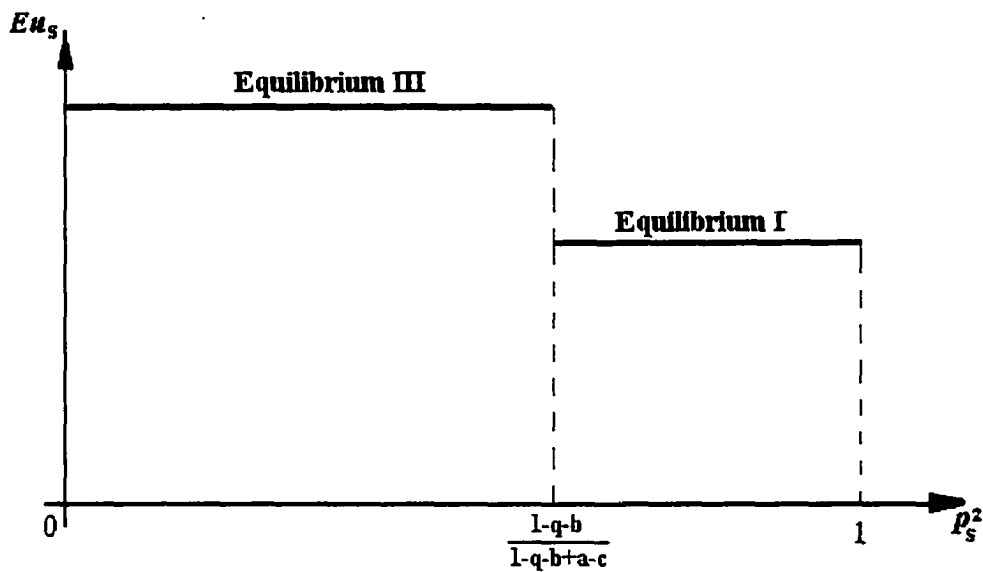


Figure 2.11. The expected utility of weak incumbent S as a function of p_S^2 .

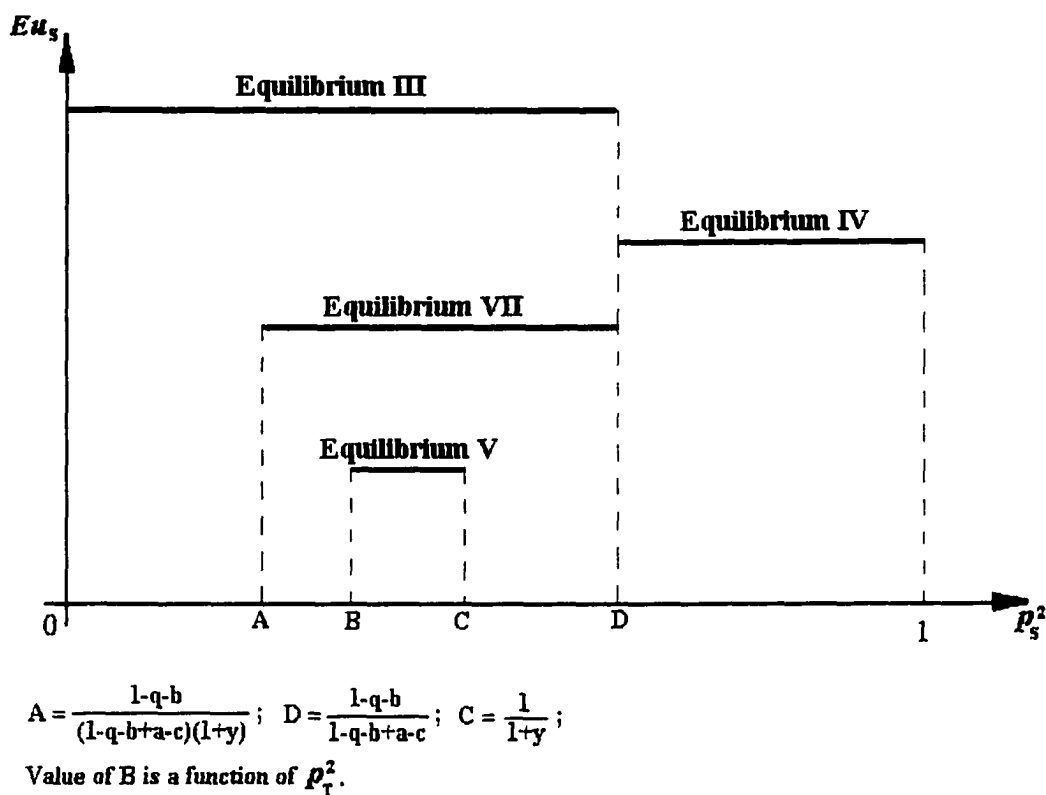


Figure 2.12. The expected utility of weak incumbent S as a function of p_s^2 .

2.3.5. Game with More than Two Periods

In subsection 2.3.3 we have completely characterized the solution to the incomplete information game with two periods. For each pair of initial reputations we have presented assessment(s) that satisfy requirements of "sensible" sequential equilibrium and corresponding equilibrium payoffs of weak incumbents. We could take these continuation payoffs as given and try to solve for the three period game. But there is a potential difficulty in doing this, since for some ranges of prior probabilities the two-period game

has multiple equilibria and, hence, non-unique continuation payoffs. Unless one has a satisfactory criterion to discard some of the equilibria, the task of finding a recursive solution for some ranges of initial reputations is practically impossible. Such a criterion cannot be found since for some ranges of prior probabilities there exist symmetric equilibria (for instance, equilibria III and IV for initial reputations satisfying $\frac{1}{1+y} < p_S^2, p_T^2 < \frac{1-q-b}{a-c+1-q-b}$). And it is impossible to distinguish between the two, unless one has a strong preference over some of the English letters. Another difficulty in solving for the game with more than two periods is that formulas for some of the equilibrium mixing probabilities in the two period game are very cumbersome. This makes it very difficult to find an analytical solution by invoking mathematical induction.

One should not interpret the discussion of the previous paragraph as a statement that for no range of initial reputations can a closed form solution be found. Instead of identifying ranges of initial reputations for which tractable solutions can be easily found we consider an infinitely repeated version of our game. We find a lower bound on equilibrium payoffs of a sufficiently patient weak incumbent in any Nash equilibrium. Our presentation here follows closely that of Fudenberg and Levine (1989) and some results are slight modifications of theirs.

In this subsection we consider the case when the game form of Figure 2.1 is played by incumbents S and T against a countably infinite sequence of different entrants. That is, we consider an infinitely repeated game where in each period $t = 0, 1, \dots$ each incumbent $i = S, T$ chooses an action a_i from the action space $A_o = \{\text{fight, accommodate}\}$ and period t 's entrant chooses an action a_1 from the action space $A_1 = \{\text{enter, stay out}\}$. Let \tilde{A}_i ($i = 0, 1$) denote the space of mixed stage game actions corresponding to the action space A_i . Mixed stage game action of incumbent $i = S, T$ is denoted by α_i . Also, let α_1 denote entrant's mixed stage game action. The information structure of this game is the same as that of Section 2.3.1. We assume that initial reputations for toughness satisfy $0 < \rho_S, \rho_T < 1$.

In this perturbed game weak incumbents S and T (as well as tough types) maximize their normalized discounted values of expected payoffs using discount factors δ_S and δ_T respectively, where $0 \leq \delta_S, \delta_T < 1$. That is, if one denotes by g_i^t weak incumbent i 's ($i = S, T$) period- t payoff, then the sequence of payoffs $g_i^1, \dots, g_i^t, \dots$ has a normalized value $(1 - \delta_i) \sum_{t=0}^{\infty} \delta_i^t g_i^t$. Let V_i denote the least payoff to weak incumbent $i = S, T$ in any Nash equilibrium of the perturbed game.

Each entrant bases entry decision on its assessment of the likelihood of the two

incumbents' actions. In our analysis we use Lemma 1 of Fudenberg and Levine (1989) which describes the process of statistical inference by the short-run players (entrants in our case) if a long-run player (there are two of them in our analysis) chooses the same action in each period (for the statement of the lemma, see Chapter 1). The application of this lemma to our model implies the following. If there is a positive initial probability of incumbent $i = S, T$ being a tough type ($\rho_i > 0$) and if incumbent i always fights then there is a fixed finite bound $k(\rho_i)$ on the number of periods in which incumbent $j \neq i$ and the entrants will believe that entry is "unlikely" to be fought. Note that this lemma does not assert that incumbent $j \neq i$ and the entrants eventually become convinced that incumbent i is a tough type if they observe that incumbent i fought in every period. Rather, incumbent i 's opponents become convinced that it will fight in the future.

Let $O(a_S, a_T, a_1)$ be the subset of $A_o \times A_o$ corresponding to the pairs of strategies (a'_S, a'_T) of incumbents S and T such that (a_S, a_T, a_1) leads to the same terminal node as (a'_S, a'_T, a_1) . Similar to Fudenberg and Levine (1989), we call these strategies *observationally equivalent* (see Chapter 1). For our game, $O(\text{accommodate, accommodate, out}) = O(\text{accommodate, fight, out}) = O(\text{fight, accommodate, out}) = O(\text{fight, fight, out}) = \{(\text{accommodate, accommodate}), (\text{accommodate, fight}), (\text{fight, fight, out})\}$

accommodate), (fight, fight)} and $O(a_S, a_T, \text{in}) = \{(a_S, a_T)\}$ for all $\{a_S, a_T\} \in A_o \times A_o$.

Let $B : \tilde{A}_o \times \tilde{A}_o \rightrightarrows \tilde{A}_1$ be the correspondence mapping pairs of mixed strategies of incumbents S and T to the best responses of the entrant. It is easy to note that $B(\text{accommodate}, \text{accommodate}) = \{\text{in}\}$ and $B(\text{accommodate}, \text{fight}) = B(\text{fight}, \text{accommodate}) = B(\text{fight}, \text{fight}) = \{\text{out}\}$.

For each a_S define

$\widetilde{WS}(a_S) = \{a_1 \mid \text{there exist no } a_T \text{ with support in } O(a_S, a_T, a_1) \text{ such that } a_1 \in B(a_S, a_T)\}$.

We prove our theorem for the case of weak incumbent S (the case of weak incumbent T is absolutely symmetric).

Theorem: There exists a $k(\rho_i)$ such that $V_i \geq \delta^{k(\rho_i)}[(1-q) - qa] + (1 - \delta^{k(\rho_i)})(-a)$.

Proof:

If $a_1 \in \widetilde{WS}(\text{fight})$, then there is a probability $\pi(a_1)$ such that a_1 is not a best response to (a_S, a_T) which places weight at least π on $O(\text{fight}, a_T, a_1)$. Let $\pi \equiv \max \pi(a_1)$. Each time entrant plays a_1 in $\widetilde{WS}(\text{fight})$, the observed outcome (fighting) will be one that was expected with probability less than π . The rest of the proof follows Lemma 1 of Fudenberg

and Levine (1989).

It follows directly from the theorem that as $\delta_i \rightarrow 1$ the right hand side of inequality (3) tends to $(1 - q) - qa$. That is, as weak incumbent becomes sufficiently patient its payoff in any Nash equilibrium exceeds $(1 - q) - qa$.

2.4. Conclusions

In this chapter we have extended the basic chain-store game to the two-incumbent case. We considered a highly stylized model where two established firms populate a finite number of geographic locations (markets). In each of these markets there is a threat of potential entry by a different competitor. Potential entrants move sequentially and choose whether to enter a market duopolized by the two incumbent firms. In case of entry, each of the two incumbents simultaneously and independently decides whether to meet entry by aggressive or a peaceful response. If a potential competitor decides to stay out, then the two incumbents split market profits. The entrant prefers to enter only if both established firms meet entry peacefully. Entry accommodation is the best short-run response for both incumbents. In the sequential game, outcomes in previous markets become known to all later players. In Section 2.2 we considered a finitely repeated game of complete information and showed that none of the potential competitors is deterred from entering in

the unique subgame perfect equilibrium.

In the model of Section 2.3 we have investigated equilibrium behavior in the incomplete information setting where each of the incumbents may have an initial reputation for being a "tough" type that will fight entry in each market along any equilibrium path. In that model each incumbent possesses as much prior information about its counterpart as the potential entrants in a sense that it assesses the same (as entrants) probability to the event that the other incumbent enjoys fighting. We completely characterized the solution of the two-period game and showed that over some range of initial reputations a weak incumbent's payoff is a globally decreasing function of its reputation. This interesting result stands in sharp contrast to the existing versions of chain-store game where high reputation for toughness always benefits a weak incumbent. In Section 2.3 we also found the lower bound on a sufficiently patient incumbent's payoff in any Nash equilibrium of the infinitely repeated version of the game. This bound is larger than the lowest payoff predicted by the Folk theorem.

In our highly stylized analysis we did not provide a sufficiently rich economic structure that would rationalize incumbents' short-run benefit from fighting. We are planning to pursue this line of research in the sequel to this work. We envision a model where firms

compete in prices and each of the established firms possesses a superior information about its private cost function and where a low-cost incumbent (synonymous to a "tough" incumbent in our analysis) prefers to set such a low price in the short-run that entry is discouraged. An important distinguishing characteristic of this analysis will be that, even if there is no entry, interaction between the incumbents is a signal of their respective costs and, hence, will affect entry decisions of subsequent rivals. This is subsumed in the present analysis for the sake of presentability.

One could also extend our analysis to more general games where two or more long-run players face a finite or infinite sequence of short-run players and where each long-run player possesses private information about its payoffs (and, hence, may try to maintain certain reputations). As our analysis suggests, the degree of common and conflicting interests of long-run players may determine equilibrium behavior when the game has a sufficiently short finite horizon. While if the horizon becomes sufficiently long one of the effects may dominate the other.

CHAPTER 3. STATE TRADING COMPANIES, TIME CONSISTENCY, IMPERFECT ENFORCEABILITY and REPUTATION⁹

3.1. Introduction

Strategic trade in international markets is important for agricultural and other basic commodities. Distribution systems for these commodities are dominated by agents that have the potential for exclusive monopoly power. State trading companies (STC) and large private firms control most of the trade volume. For agriculture in particular, the seasonal nature of the production process compared to a relatively constant demand for the commodities brings into play as well concepts of timing in trade contracting and time consistent behavior.

Some of the earliest papers that investigate issues of time consistency in trade are Lapan (1988), Maskin and Newbery (1990) and Staiger and Tabellini (1987). Melkonian and Johnson (1996) have explored a model in which a STC, which has monopsony power, cannot credibly commit to a particular policy or contract. An annual trading cycle was considered. In the sequence of economic decisions, the STC moves first and announces a planned level of import. Producers in the exporting countries make their decisions on the allocation of the more fixed inputs (e.g., land) based on related price expectations. However, before they make decisions on the allocation of the more variable inputs (e.g.,

labor or fertilizer), the STC has the opportunity to revise the announced level of imports. Then, the labor or variable input allocation decisions of the producers are made, given the revised (and predetermined) level of imports and the previous allocation of the more fixed inputs. Finally, trade takes place.

A standard monopsony argument can be used to obtain the optimal level of import, if the STC can commit itself to the announced import level. But, when the STC cannot be held to the precommitted or the ex ante optimal level of import, it has an incentive to set a lower ex post level, once the land allocation decision has been made (the STC will face an ex post supply that is less elastic than the ex ante supply). In standard terminology, the ex ante optimal level of import is not time-consistent. If foreign producers are assumed to know the rule used in setting ex post level of import, they will use this information when making their land allocation decisions. We showed that both the importer (STC) and the exporting countries are worse off as a result of inability of the importer to precommit to the optimal level of import. It is shown that forward contracts allow one to support the ex ante optimal level of import as a time-consistent equilibrium. We also considered the case when the importer is better informed than the exporter and showed that under some scenarios the importer cannot benefit from the superiority of its information.

In this paper, we use a standard game theoretic formulation to model the strategic behavior we have described, and to explain apparent anomalies in trade performance. The same game theoretic formulations can be used to examine the impact of mechanisms with the potential for dealing with time inconsistency. In particular, precommitment penalties that might be enforced by bonding or other types of trade management systems are evaluated. These mechanisms can result in Pareto superior trade (and investment) patterns. An alternative is to consider the trading strategies in a sequential game context. The implications of signaling and reputation effects are at issue in this sequential or multi-stage trading context. This formulation opens the possibility of behavior that avoids the suboptimal elements of the single period game. As well, the strategies that emerge suggest alternative mechanisms that dominate the unregulated outcomes for the simple sequential game, even if they involve contracts that are imperfectly enforceable.

3.2. The Perfect Information Game

We consider a sequential game between two players. The game has three periods. In the first period, player 1 (the STC) announces the policy she intends to implement in the third period. That is, player 1 makes one of two announcements: "benevolent" or "nonbenevolent". This announcement becomes known to player 2 (the STC's trading

partner). After either of the two announcements is made, players 1 and 2 play the game depicted in Figure 3.1 (numbers at the decision nodes in the game tree represent the player whose turn it is to move; the first of the pair of numbers at terminal nodes represents payoff of player 1, and the second, the payoff of player 2). As will be clear, the "cheap talk" of player 1 does not affect the future strategic interaction between the players.

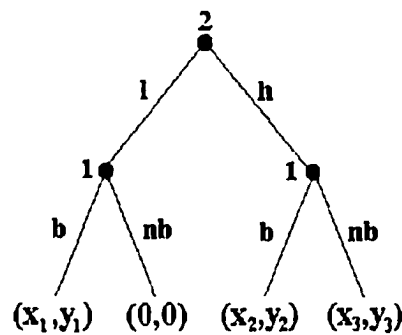


Figure 3.1. Investment choice - policy implementation sequence.

Specifically, in the subgame following player 1's initial announcement, player 2 moves and chooses one of two levels of investment, "high" or "low" (h and l in Figure 3.1). After observing the level of investment, player 1 makes her choice of policy to be implemented: "benevolent" or "nonbenevolent" (b and nb in Figure 3.1). The complete (with the policy announcement) game form and the payoffs to the players, depending on the history of the game, are shown in Figure 3.2. Note that the two proper subgames starting at the nodes where it is player 2's turn to move are equivalent (the game forms and the payoffs are the

same). This reflects the noncredible cheap talk announcement of player 1.

Now, consider in more detail payoffs to the players for different strategy profiles. For both of the possible levels of investment selected by player 2, player 1's payoff is higher if she chooses to implement nonbenevolent compared to the benevolent policy; $x_1 < 0$ and $x_2 < x_3$. In contrast, player 2's payoff, given that the investment decision has been made, is higher if the benevolent policy is implemented by player 1; $y_1 > 0$ and $y_2 > y_3$. If an investment decision is followed by a benevolent policy, player 2's payoff is higher when he chooses high; $y_2 > y_1$. While if an investment decision is followed by a non-benevolent policy, player 2's payoff is higher when he chooses low; $y_3 < 0$. Player 1's payoff is higher when high investment is followed by a benevolent policy than in the case when low is followed by non-benevolent; $x_2 > 0$. We also assume that $x_3 + x_1 > 0$. With this assumption, the sign restrictions on the payoffs can be easily summarized: $x_1 < 0$, $0 < x_2 < x_3$, and $x_3 + x_1 > 0$; $y_3 < 0$ and $0 < y_1 < y_2$.

Consider either of the two proper subgames following initial policy announcement, i.e. the game depicted in Figure 3.1. If player 2 chooses low, player 1 chooses between x_1 if she plays benevolent and 0 if she plays nonbenevolent. Thus, obviously player 1 will play nonbenevolent. The same reasoning leads us to conclude that when high investment is

chosen by player 2, nonbenevolent policy will be implemented (in that case, player 1 chooses between x_2 and x_3). That is, nonbenevolent is the ex post (after the investment decision) optimal policy. Anticipating that either level of investment will be followed by implementation of nonbenevolent policy, player 2, when making his investment decision, chooses between 0 if he plays low and y_3 if he plays high. Obviously, he will choose the low investment. When making her initial announcement, player 1, anticipating that she and her opponent will play optimally at later nodes, chooses between 0 if she announces benevolent and 0 if nonbenevolent, and she is indifferent between the two.

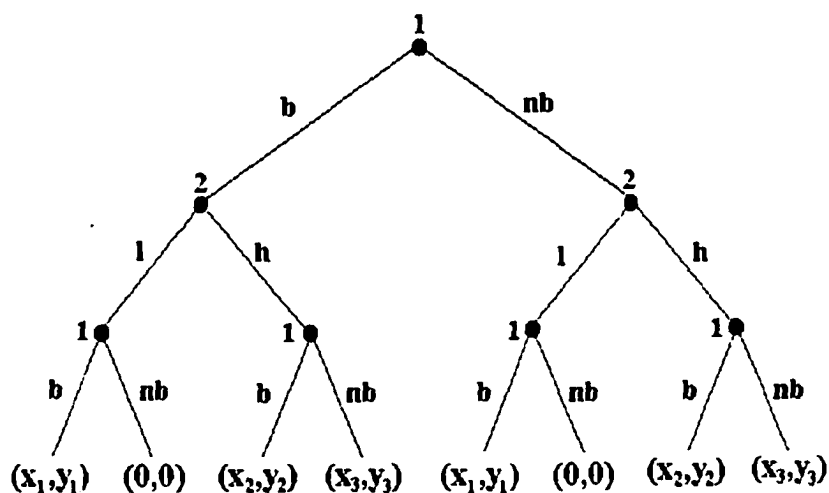


Figure 3.2. The Basic Model.

This argument is just a simple application of backward induction to the solution of a game of perfect information.¹⁰ Thus, we have shown that this game has two subgame perfect equilibria: (1) player 1 initially announces benevolent and implements nonbenevolent policy for all levels of investment, and for either of the initial announcements, player 2 chooses low investment no matter what the initial announcement; and (2) player 1 initially announces non-benevolent, and the actions at all other information sets are the same as for the first strategy profile (the second equilibrium differs from the first only by the move of player 1 at her first information set). The equilibria payoffs of both players are the same for both of the strategy profiles. Both yield a payoff of 0.

Now, suppose that there is a mechanism that allows player 1 to credibly precommit to a policy or announcement. One possible mechanism can be described as follows: suppose that before the game player 1 signs a perfectly enforceable (binding) agreement saying that she is going to burn \$ c ($c > x_3$) if she does not implement the announced policy. The tree for this game is presented in Figure 3.3. Note that the game form is unchanged and only the payoffs to player 1 at the terminal nodes, corresponding to histories where the announced and implemented policies differ, have been modified (for example, announcing benevolent and implementing nonbenevolent policy). There is also another interpretation of

the extensive form game presented in Figure 3.3. Suppose that player 1 is known for sure to be "commitment" (or an "honest") type, that is a player who gets a very high negative payoff ($-c$) from renegeing on her announcement in the first period. In other words, player 1 incurs very high cost from being inconsistent.

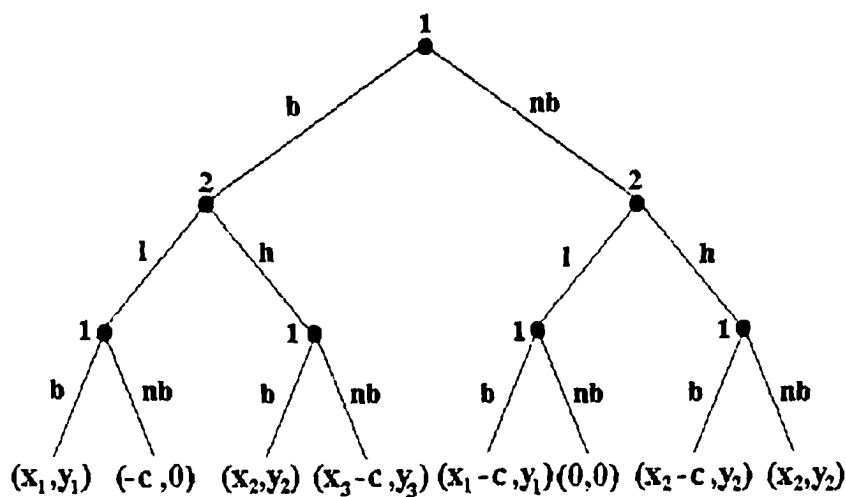


Figure 3.3. The game with perfectly enforceable commitment mechanism.

We solve the game with this commitment mechanism. Consider the proper subgame starting with the node following player 1's initial announcement of benevolence. If player 2 chooses low, player 1 chooses between x_1 if she plays benevolent and $(0 - c)$ if she plays non-benevolent. Thus, obviously player 1 will choose benevolent. The same reasoning leads us to conclude that benevolent policy will be implemented by player 1 when high

investment is chosen by player 2 (in this case, player 1 would choose between x_2 and $(x_3 - c)$). Hence, when player 2 makes his investment decision after the announcement of benevolence and anticipates the above characterized response (implementation of the benevolent policy), he chooses between y_1 if he chooses low and y_2 if he chooses high. Surely, he will choose high investment.

Applying the same reasoning (backward induction) to the proper subgame starting with the node following a nonbenevolent announcement, we find that in the subgame either investment decision will be followed by the implementation of the nonbenevolent policy and player 1 (anticipating the nonbenevolent response) will choose low investment. Thus, rolling back the payoffs of the players (when the strategies obtained by backward induction are followed) to the nodes that follow initial announcement, we see that player 1 chooses between x_2 if she chooses benevolent announcement and 0 if she chooses nonbenevolent. And hence, she will choose benevolent. Thus, we have shown that this game has unique subgame perfect equilibrium, where: player 1 announces benevolent in period 1 and at the node following a particular announcement, she implements the policy that was announced; player 2 chooses high if benevolent was announced and low if nonbenevolent. The subgame perfect equilibrium payoffs to the players are (x_2, y_2) .

We could consider yet a larger game where player 1 chooses in period 0 whether to play the game described in Figure 3.2 or the game of Figure 3.3 (the first interpretation (burning money) will be chosen for Figure 3.2, since it is not sensible to assume that people choose to be honest or not) and that decision is made known to player 1. In this game player 1 will choose the game with the precommitment mechanism. There are two subgame perfect equilibria of this game: player 1 in period 0 chooses to play the game with commitment mechanism and the choices of both players after the choice in period 0 are exactly the same as equilibria strategy profiles of players 1 and 2 for the games presented in Figures 3.2 and 3.3.

The multiplicity of equilibria arises from the fact that the proper subgame following the choice of player 1 not to choose a commitment mechanism has two subgame perfect equilibria (as described above). The payoffs for both the equilibria described are (x_2, y_2) . Thus, if player 1 has an option of a commitment mechanism, she will use that option and the payoffs of both players are higher in the game when this option is available $(x_2 > 0, y_2 > 0)$, as compared with the case when it is not.

3.3. The Finitely Repeated Version of the Game Without Commitment

Consider the strategic situation when player 1 does not have access to the commitment mechanism and there are finitely many trading cycles after policy announcement is made. Again, there are two players. The game has $N + 1$ stages (N is a positive integer). We index time backwards; i.e., the first stage of the game is $N + 1$, second N , and so on. At the stage $N + 1$, player 1 makes one of two announcements, benevolent or nonbenevolent. After the announcement is made and observed by player 2, the game depicted in Figure 3.1 is played N times. We assume that the announcement in stage $N + 1$ is noncredible in a sense that it does not affect the moves available to the players and the payoffs for each strategy profile (the two proper subgames, following the announcement, are exactly the same).

The sign restrictions for different strategy profiles are the same as in the previous section.¹¹ The payoffs of players 1 and 2 are the sums (undiscounted) of their respective payoffs in the stages of the game. This is a finite game with perfect information, which again can be solved by application of backward induction. First, let us conjecture how the game might progress. We might expect that player 1 will announce benevolent in stage $N + 1$, and then will implement benevolent policy no matter what the level of investment, to convince player 2 that she is honest (committed to her announcement), and that she will

continue to implement the announced policy. In other words, player 1 will try to establish a reputation for being a "commitment" type. Observing this kind of choice by player 1, player 2 will become convinced that player 1 will keep to the announced policy (benevolent) and will choose high investment level. Thus, we would expect benevolent policy to be implemented and high investment for player 2 in the first stages of the game. However, we also expect that this kind of behavior will not be observable at very late stages in the game, since later in the game there is not much benefit from demonstrating commitment (the reputation value is low). That is, we would expect nonbenevolent policies to be implemented at later stages of the game. Though the behavior we have just described seems likely to develop and is very intuitive, the game theoretic prediction discards it.

To solve for the subgame perfect equilibrium, consider a proper subgame starting with the node following either of the two announcements. Within that subgame consider the last stage. In this stage, player 1 will implement the nonbenevolent policy for both levels of investment by player 2, since there are no gains left to maintain her reputation. Anticipating this response, player 2 will choose low investment. In the next to the last stage, player 1 will implement nonbenevolent policy whatever level of investment, since it is more beneficial in the short-run and does not affect the play in the last stage. Anticipating this,

player 2 will choose low investment in the next to the last stage. Carrying this argument to the beginning of the subgame, we find that player 2 chooses low investment in all stages, and player 1 always implements the nonbenevolent policy. Again, as in the Section 2, the game has two subgame perfect equilibria, which differ only by the initial announcement and have low investment and nonbenevolent policy implementation for each possible history. The equilibrium payoffs for both strategy profiles are $(0, 0)$.

Now, suppose we consider the game which consists of K stages (K is finite positive integer), where each stage is exactly the game described above (announcement of a policy followed by N repetitions of the game in Figure 3.1). Again, we reach conclusion that the subgame perfect equilibria will have low investment and nonbenevolent policies implemented for all information sets (Recall that all information sets are singletons in this game). Thus, even in the case when there are multiple (and finite) subperiods of announcements followed by investment choice-implementation sequences, the reputation effect does not "come alive". In summary, although it is intuitive and plausible for player 1 to try to maintain a reputation for being a commitment type by implementing the policy announced to persuade player 2 to make a high investment, in none of the subgame perfect equilibria is this an optimal strategy.

3.4. Imperfect Enforceability, Pooling and Reputation Effects

We have shown that, for the basic game, if there is an option of commitment mechanism that is also perfectly enforceable then high investment is chosen and the ex ante optimal policy is implemented. This results in a payoff increase for both players. Then, we turned to the case when the trading game is repeated finitely many times, but with the commitment mechanism absent. We showed that no matter how many (finite) times the game is to be played, reputation effects do not come alive and the payoffs to the players are the same as in the one-period version of the game $(0, 0)$.

In this section we investigate a model with an imperfectly enforceable commitment mechanism. As before, the commitment mechanism obliges the party (player 1), reneging on announcement, to pay a penalty of $\$c$.

First, consider the game where announcement of the policy is made followed by N repetitions of investment choice-implementation sequence. As previously, we assume that the game has two players. Before the game is played, player 1 makes a contract with the third party, which obliges her to implement the announced policy. The information, on whether this commitment contract is going to be honored is, however, the private information of player 1. That is, the commitment contract with the third party is

"imperfectly enforceable." We assume that player 2 has prior probability belief ρ that player 1's payoffs are the same as in Figure 3.3, each time she reneges on her announcement. That is, ρ is the probability of player 1's being the commitment type. With probability $(1 - \rho)$ player 1's payoffs are as in Figure 3.2. That is, $(1 - \rho)$ is the probability of player 1 being a noncommitment type, for whom the policy announcement is a cheap talk. In other words, player 2 is uncertain about player 1's cost of renegeing on her announcement. This is a game of incomplete information (Harsanyi (1967-68)), which can be transformed into a game of imperfect information where nature moves first and chooses player 1's payoff structure, player 1 observes nature's move but player 2 does not. The game with a single investment choice-policy implementation stage is depicted in Figure 3.4. We denote by θ_c the commitment type player 1, and by θ_n the noncommitment type.

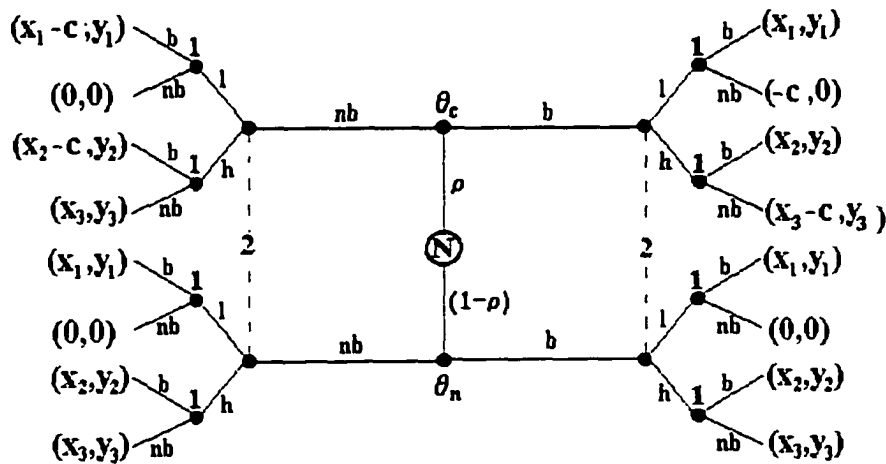


Figure 3.4. The game with imperfect information.

We first analyze the game depicted in Figure 3.4 and then move on to solve the case of an arbitrary number (finite) of investment choice-policy implementation stages. Note, that for the commitment type it is a strictly dominated strategy to renege on the announcement made in the first stage. Thus, we can eliminate the strategies where commitment type implements policy different than the one announced as possibilities for equilibrium behavior. Given an initial announcement and either investment level, implementation of the nonbenevolent policy is optimal for the noncommitment type (by application of backward induction). In other words, it is a strictly dominated continuation strategy for the noncommitment type to implement benevolent policy at any information set. Eliminating the strictly dominated strategies for different player 1 types, it is easy to show that the best response for player 2 at the information set following nonbenevolent announcement is to choose low investment for any beliefs about the type of player 1 (commitment type implements the policy announced, and the noncommitment type always implements nonbenevolent policy).

Thus, the choice of the low investment after the announcement of the nonbenevolent policy is justified. Given these rounds of elimination of the strictly dominated continuation strategies, the strategies in which the noncommitment type of player 1 announces the

nonbenevolent policy are weakly dominated by the strategy where she announces the benevolent and implements the nonbenevolent policy for either investment level. To eliminate weakly dominated strategies, we invoke Kohlberg and Mertens' (1986) requirement that the equilibria set be stable. Elimination of weakly dominated strategies corresponds to setting their admissibility condition. The above described elimination of dominated strategies has allowed us to reduce the set of possible modes of behavior for the two players. Now, we can move on to finding the Nash equilibria of the game in Figure 3.4.

Note, that no matter what the value of ρ in the interval $[0, 1)$, the following strategy profile is a Nash equilibrium of the game in Figure 3.4:

i) the commitment type of player 1 announces nonbenevolent policy and at the information sets following a particular announcement implements that promise; the noncommitment type announces benevolent policy and implements nonbenevolent policy given either level of investment and either announcement;

ii) player 2 chooses low investment following either announcement by player 1.

It is easy to note that the equilibrium just described is sequential if we specify beliefs at the two information sets of player 2 using Bayes' law to update prior beliefs, given player

1's equilibrium strategy. We denote this equilibrium by (*). Using the terminology of signaling games, this equilibrium is separating in a sense that the signal (the announcement) sent by player 1 in the first stage reveals her type. This equilibrium set (singleton) is also stable. To find all possible equilibria, we consider two possible cases differentiated by the magnitude of ρ and relative values of player 2's payoffs for different strategy profiles:

$$(3.1) \rho \geq \frac{-y_1}{y_2 - y_1 - y_3}$$

(the case in which the relative likelihood of player 1s being a commitment type is high)

Consider the strategy of player 1 where her both types announce benevolent with probability 1. Then Bayes' updating yields posterior which is equal to prior beliefs about type of player 1. That is, probability that player 1 is of commitment type, given that both types announce benevolent with probability one, is equal to ρ . Then, given updated beliefs and player 1's optimal strategies following the investment decision, player 2's expected payoff from high investment is equal to $\rho y_2 + (1 - \rho)y_3$ and from low investment to $\rho y_1 + (1 - \rho)0$. Hence, if the inequality (3.1) is satisfied and player 1 uses the strategy described above, player 2 will choose high investment. Thus, following strategy profiles and beliefs of player 2 about player 1 constitute a sequential equilibrium (also stable as a

set):

i) Both player 1 types announce benevolent policy in the first stage; at the information sets following a particular announcement commitment type of player 1 implements the announced policy (implementation of benevolent policy after announcement of benevolence, and similarly for non-benevolent policy), the noncommitment type chooses to implement nonbenevolent strategy at all of her information sets:

ii) Player 2 chooses low investment at the information set following nonbenevolent announcement, and chooses high investment at the information set following benevolent announcement;

iii) At the information set following the benevolent announcement, the probability (posterior) that player 1 is of commitment type is equal to the prior ρ (Bayesian updating is invoked using equilibrium strategy of player 1); at the information set following the nonbenevolent announcement the posterior probability that player 1 is of commitment type is equal to q , where q is any real number in the segment $[0, 1]$.

It is easy to observe that (strategy profile, beliefs) pair is consistent and sequentially rational (requirements of a sequential equilibrium). This is a pooling equilibrium in the sense that both player 1 types choose to send the same signal (announcement of

benevolence). The equilibrium payoffs of the noncommitment type of player 1 and of player 2 are x_3 and $\rho y_2 + (1 - \rho)y_3$ respectively. Payoffs of both players are higher than in the case when player 1 does not have a reputation for being a commitment type (that is, when $\rho = 0$).

Recall, that for the values of ρ satisfying inequality (1) the (strategy profile, beliefs) pair (*) also represents a sequential equilibrium. But, the outcome payoffs of both players for this equilibrium are Pareto dominated by the equilibrium payoffs for the just described pooling equilibrium. We use the coalition-proof Nash equilibrium¹² concept of Bernheim, et al. (1987) to discard the equilibrium (*). We argue that the pooling equilibrium is more likely to be played than separating one, because if preplay communication were possible then both players would have an incentive to agree to play pooling equilibrium (which is a self-enforcing mode of behavior).

When the initial reputation for credibility is low

$$(3.2) \rho < \frac{-y_3}{y_2 - y_1 - y_3}$$

the only sequential equilibrium, which also survives the elimination of dominated (strictly as well as weakly) strategies, is separating where the noncommitment type chooses the benevolent announcement and the commitment type chooses the

nonbenevolent announcement (that is, equilibrium (*)). The equilibrium payoffs of both player 1 types and player 2 are equal to zero, i.e. they are the same as in the case when player does not have a reputation for being commitment type.

Our conclusions can be easily summarized: For the game where there is only one investment choice-policy implementation stage after the policy announcement, if the prior probability of player 1 being a commitment type is not sufficiently large the resulting equilibrium is the one where the different player 1 types separate in the first stage (announcing different policies) and the equilibrium payoffs of both players are equal to zero (same as when there is no reputation for being a commitment type). When reputation of being commitment type is sufficiently large, the pooling equilibrium is the only one that survives all the criteria that we have imposed and payoffs of both players are higher than in the case where reputation is absent.

Now, we consider a more general game in that we allow the announcement stage to be followed by arbitrary but finite number of investment choice-policy implementation stages. Again, there are two players in the game: player 1 and player 2. The game has $N + 1$ stages (N is a positive integer). As previously, we index time backwards. Again, the first stage of the game is $N + 1$, second N , etc. At the stage $N + 1$ player 1 makes one of two

announcements: benevolent or nonbenevolent. After the announcement is made and observed by player 2, the investment choice-policy implementation game form is played N times. But in contrast to the game described in the third section, player 2 is uncertain about the payoffs of player 1, and holds a prior probability ρ that player 1 incurs a cost each time she (player 1) reneges on the announcement (her payoffs are as in Figure 3.3). With probability $(1 - \rho)$ the payoffs of player 1 for each stage game are those given in Figure 3.2. That is, $(1 - \rho)$ is the probability of player 1 being a noncommitment type. The payoffs of players 1 and 2 are the expected sums (undiscounted) of their respective payoffs in the stages of the game. In the following we will be interested in the payoffs for the noncommitment type of player 1 and payoffs of player 2.

Before trying to determine how this game will be played, note that commitment type of player 1 will implement the announced policy along each equilibrium path of this game. That is, there are two possibilities for commitment type's behavior in equilibrium: announce and then in all stages implement benevolent policy, or announce and then in all stages execute nonbenevolent policy. Note, that our model differs from other reputation models since we allow the type, whose reputation is to be maintained, to be strategic, i.e. that player type is not restricted to a single strategy along the equilibrium path.

Let h_j denote the history of the game up to stage j . Player 2 will update his beliefs about the player 1 type conditional on the previous moves of both players (h_j). We denote these beliefs at the beginning of stage j by p_j . As indicated above, p_j is a function of h_j . For any value of ρ (the initial reputation for being the commitment type) between 0 and 1, the following strategy profile and belief structure constitute a sequential equilibrium:

i) the commitment type player 1 announces nonbenevolent policy and at the information sets following a particular announcement implements the promised policy; the noncommitment type announces benevolent policy and implements nonbenevolent at all of her information sets;

ii) player 2 chooses low at all of his information sets;

iii) at all information sets following the announcement of nonbenevolent policy, player 2's belief that player 1 is commitment type is equal to 1; at all information sets following the announcement of benevolent strategy, player 2's belief that player 1 is commitment type is equal to 0.

It is possible to show that the equilibria set (a singleton) just described is stable. Using terminology of signaling games, this is a separating equilibrium, because the signal (in this case, the signal is an announcement of policy) sent in the first stage of the game reveals

sender's type. Subsequently, we term this a separating equilibrium.

This game also has another sequential equilibrium for large enough ρ and/or N . To solve for this equilibrium of the finite game of imperfect information, we first find the sequential equilibrium when there is only one investment choice-policy implementation stage after the announcement. Then we solve when there are two stages, and then invoke mathematical induction to find the solution for arbitrary (finite) number of stages. The following strategies and beliefs constitute a sequential equilibrium¹³ if:

$$\rho \geq \left(\frac{-y_3}{y_2 - y_1 - y_3} \right)^N$$

Beliefs:

i) $p_{N-1} = \rho$;

ii) For any $j \leq N$, probability that player 1 is of commitment type at each information set of player 2 following announcement of nonbenevolent policy is equal to one;

iii) $p_N = \rho$ at the information set following benevolent announcement;

iv) If the history of the game up to stage $j < N$ includes benevolent announcement and any instance of implementation of nonbenevolent policy then $p_j = 0$;¹⁴

v) If $p_{j-1} = 0$ then $p_j = 0$;

vi) If benevolent policy was initially announced, $j > 1$, $p_j < \left(\frac{-y_3}{y_2 - y_1 - y_3} \right)^{j-1}$ and

the investment decision is followed by implementation of the benevolent policy in stage j ,

$$\text{then } p_{j-1} = \left(\frac{-y_3}{y_2 - y_1 - y_3} \right)^{j-1};$$

vii) If benevolent policy was initially announced, $j > 1$, $p_j \geq \left(\frac{-y_3}{y_2 - y_1 - y_3} \right)^{j-1}$ and

the investment decision is followed by implementation of the benevolent policy in stage j ,

$$\text{then } p_{j-1} = p_j.$$

The *strategy of player 1* is described as follows:

i) the commitment type announces benevolent policy in stage $N + 1$ and at information sets following an announcement of benevolence she implements benevolent, at information sets following nonbenevolent she implements nonbenevolent;

ii) the noncommitment type announces benevolent policy in stage $N + 1$: at all information sets following a nonbenevolent announcement she implements nonbenevolent policy; the choice of the noncommitment type at the information sets following benevolent announcement (nodes following both high and low investment) depends on p_j and j :

If $j = 1$ then she implements the nonbenevolent policy, given any investment choice:

If $j > 1$ and $p_j \geq \left(\frac{-y_3}{y_2 - y_1 - y_3} \right)^{j-1}$ then she implements the benevolent policy;

If $j > 1$ and $p_j < \left(\frac{-y_3}{y_2 - y_1 - y_3} \right)^{j-1}$ then she implements the benevolent policy with

probability $\frac{(A-1)p_j}{(1-p_j)}$ (where $A = \frac{y_2 - y_1 - y_3}{-y_3}$) and the nonbenevolent with complementary probability.

The *strategy of player 2* can be described using related conditions:

Player 2 chooses high investment in stage j if $p_j > \left(\frac{-y_3}{y_2 - y_1 - y_3}\right)^j$, he chooses low investment if $p_j < \left(\frac{-y_3}{y_2 - y_1 - y_3}\right)^j$. If $p_j = \left(\frac{-y_3}{y_2 - y_1 - y_3}\right)^j$, then player 2 randomizes between his choices, playing high with probability $\frac{x_3 - x_2}{x_3}$ in case when high investment was chosen in the previous stage and with probability $\frac{-x_1}{x_3}$ if low investment was chosen in the previous stage.

The nature of the equilibrium is the following: for every $\rho > 0$ there is a number $n(\rho)$ such that, if there are more than $n(\rho)$ investment choice-policy implementation sequences remaining to be played, the noncommitment type will implement the benevolent policy and player 2 will choose high investment. The noncommitment type chooses nonbenevolent in the last stage and mixes between benevolent and nonbenevolent in stages $n(\rho), \dots, 2$. Accordingly, player 2 mixes between high and low or chooses high with probability, which depends on the relative magnitudes of ρ and the players' payoffs. Thus, in each of the stages $n(\rho), \dots, 1$, reputation breaks down with positive probability. Reputation breaks

down in the later stages, since long-run value from having reputation for commitment is outweighed by the (opportunity) cost of pretending to be a commitment type. Note, that for large enough N , payoffs of both players converge to 3.

For large enough ρ and/or N this equilibrium outcome dominates the separating outcome. Hence, applying the coalition-proof Nash equilibrium concept, we discard the separating equilibrium.

If we consider a larger game consisting of K stages, where each stage is the game just described, then the equilibrium (that survives the refinements we pose) will have the following properties: In $K - 1$ first stages, both types of player 1 announce the benevolent policy and subsequently implement benevolent in all periods of the stage; player 2 chooses high investment in all periods of the first $K - 1$ stages; the play in the last stage is the same as for the single stage game described above (that is, the game with announcement followed by N repetitions of investment choice-policy implementation sequence).

3.5 Concluding Comments

We have used concepts of modern game theory to treat time inconsistency issues associated with strategic trade. The results are particularly applicable to trade in commodities with production periods that are lengthy, as in agriculture. A game with a

sequence of decisions is envisioned, in which an importing firm or country might announce its planned level of import. The producer or suppliers then "invest" by perhaps allocating land to the commodity to be exported. The importer may then change the first decision or be credible and follow through as planned. Clearly, the investment decision of the exporter will be different depending on whether or not the importer's announcement is believed (high or low in our stylized model). From the results on time consistency, we know that revision or noncommitment can lead to an outcome that is suboptimal compared to the initial (ex ante) decision on announcement. Mechanisms are then explored that impose a "cost" for the failure to honor the initial commitment. An example of such a mechanism in actual trade might be the posting of a bond by one of the parties. Both parties (the STC and its trading partner) gain, if these mechanisms are perfectly enforceable.

We investigated the game in which the parties are assumed to trade for more than one period, and where enforcement mechanisms are absent. Results show that the optimality problems are similar to those for the single stage game, in terms of their time inconsistency implications. Then the sequence of trading decisions or games is examined in which the commitment mechanism is present, but imperfectly enforceable (we represent this by assuming that information on whether a commitment contract will be honored is private

information of the STC). This opens the signaling possibilities and the use of experience with previous trading actions to establish reputations. We show that if the parties trade for sufficiently many periods and/or the initial reputation for being commitment type is sufficiently high, then reputation effects dominate the play of the game. Both players gain compared to the case in which the commitment mechanisms are absent. The major applied implication is for a role of some type of authority that could "enforce" announced intentions.

Finally, we observe that the model of strategic trade has required innovations in the game theoretic formulation. The most important of these follow from the "personality" of the "commitment" type.

The model provides an important context for exploring impacts of signaling, reputation, and third party interventions that approximate the institutions and authorities that govern international trade.

APPENDIX I. BASIC GAME THEORETIC CONCEPTS

For the sake of completeness, in this appendix we discuss some equilibrium concepts frequently employed in game theory. All definitions can be found in Fudenberg and Tirole (1991a) or in any other Game Theory textbook.

Definition: A *strategy profile* is a vector consisting of mixed strategies for all players.

Definition: A strategy profile is a *Nash equilibrium* if each player's strategy is an optimal response to the opponents' strategies.

Definition: A *subgame* is a restriction of an extensive form game to a collection of nodes satisfying closure under succession and preservation of information sets. (That is, all nodes in an information set followed by all successors of all of its nodes).

Definition: A *proper subgame* is a restriction of an extensive form game to an information set, consisting of a single node, with all of its successors.

Definition: A strategy profile is a *subgame perfect Nash equilibrium* if the restriction of the strategy profile to every proper subgame is a Nash equilibrium.

Definition: A *Bayesian Nash equilibrium* is a strategy profile such that each player's strategy maximizes his expected payoff given prior beliefs about opponents' private information and their strategies.

The equilibrium concept, usually employed in reputation models, is *sequential*

equilibrium (Kreps and Wilson (1982a)). This equilibrium is not simply a strategy profile, but a pair of two types of beliefs for each player: beliefs about where in the game tree he is for each possible history of the game, and beliefs about how other players will play. The system of beliefs is a sequence of functions (each function corresponding to a particular information set) from the nodes in the information set to $[0, 1]$ segment. That is, a system of beliefs reflects what a player believes about where he is in the information set (where it is his turn to move) if that information set is reached. An assessment is a pair (strategy profile, system of beliefs). An assessment is said to be *consistent* if it can be derived from Bayes' law using arbitrary small trembles from the strategy profile. An assessment is *sequentially rational* if behavior of each player satisfies the following property: for each player, his strategy starting from any information set is optimal, given his beliefs about where in the information set he is and given that the other players play according to their strategies from that information set on. An assessment, which satisfies consistency and sequential rationality properties, is sequential equilibrium.

Another equilibrium concept, that in certain contexts is easier to apply than sequential equilibrium, is a *Perfect Bayesian equilibrium*. Fudenberg and Tirole (1991b) give a formal presentation of the concept. We only give their definition for the multi-stage games

with observed actions and independent prior distributions of players' private information.

An assessment is a Perfect Bayesian equilibrium if:

1) beliefs are updated using Bayes' law whenever possible (that is, for all information sets reached with positive probability along the equilibrium path);

2) beliefs satisfy a no-signaling-what-you-don't-know condition which requires that i) posterior beliefs about players' types are distributed independently for any history of the game and ii) beliefs about type of a player at the beginning of period $(t + 1)$ depend only on the history of the game up to stage t and that player's period- t action;

3) The strategies are a Bayesian Nash equilibrium for each subgame (where the probability distribution over the nodes for each information set are defined using the beliefs of the assessment).

Each Perfect Bayesian equilibrium is sequential, while the converse holds only in the case when each player has no more than two types and/or the game has no more than two stages.

Definition: A strategy profile is an *extensive-form trembling hand perfect equilibrium* if it is a limit of ϵ -constrained equilibria as trembles ϵ converge to 0.

Let $e_i \in \tilde{A}_i$ for each i denote a completely mixed action, that is, e_i puts a strictly

positive weight on each action $a_i \in A_i$.

Definition: The strategy profile $(\tilde{\sigma}_1, \tilde{\sigma}_2) \in \Sigma_1 \times \Sigma_2$ is an ϵ -constrained equilibrium relative to (e_1, e_2) if there exists a strategy profile (σ_1, σ_2) such that for each history $h \in H$,

i) $\tilde{\sigma}_1(\omega)(h) = (1 - \epsilon)\sigma_1(\omega)(h) + \epsilon e_1$ for each $\omega \in \Omega$;

ii) $\tilde{\sigma}_2(h) = (1 - \epsilon)\sigma_2(h) + \epsilon e_2$;

iii) $\sigma_1(\omega_o)$ is a best response to $\tilde{\sigma}_2$;

iv) σ_2 is a best response to $\tilde{\sigma}_1$.

APPENDIX II. SOLUTION OF THE TWO-PERIOD GAME

In this Appendix we prove that assessments characterized in Section 2.3.3 of this chapter form a sequential equilibrium of the two-stage game over a relevant range of initial reputations. Our discussion also shows that the equilibria presented in this Appendix exhaust the list of all "sensible" sequential equilibria. That is, there is no assessment other than the ones described here such that it is sequentially rational, consistent and "sensible" where "sensibility" means an additional restriction on off-equilibrium beliefs. We begin by discussing the properties possessed by all the equilibria including the "sensibility" requirement. Then we list equilibria assessments and the corresponding ranges of initial reputations over which these assessments satisfy requirements of sequential equilibrium. There we prove that the hypothesized assessments are a "sensible" sequential equilibrium if only if initial reputations of the two incumbents belong to the specified range. Finally, we discuss the uniqueness properties of different equilibria where we show that for some ranges of initial reputations there is a multiplicity of equilibria.

To find players' optimal strategies we utilize an equilibrium concept stronger than sequential equilibrium in that we impose additional constraints on the players' beliefs following off-equilibrium play. In particular, we assume that in the event of accommodation by any incumbent, its reputation of being tough is updated to zero by other

players. This restriction is a version of Cho and Kreps' (1990) stability and Crawford and Sobel's (1982) intuitive criterion. Given this restriction on beliefs tough incumbents will fight at any information set of the game. Also, in any sequential equilibrium of the game both weak incumbents accommodate in period 1, tough entrants always enter, and weak entrant 1 enters if and only if the pair of updated reputations (p_S^1, p_T^1) lies in the area V . Thus, when characterizing equilibria we focus on the actions of weak players in period 2 and of weak entrant in period 1.

Now we present equilibrium assessments and corresponding sets of initial reputations.

Equilibrium I:

Conditions on *initial reputations*: $p_S^2, p_T^2 > \frac{1-q-b}{a-c+1-q-b}$.

The set of probabilities (p_S^2, p_T^2) satisfying these inequalities is the shaded area of Figure 2.3 which is depicted for the case when $\frac{1-q-b}{a-c+1-q-b} > \frac{1}{1+y}$ (which holds under our assumption on payoffs of weak incumbents and prior probability of an entrant being tough). This shaded area is the square with endpoints $(\frac{1-q-b}{a-c+1-q-b}, \frac{1-q-b}{a-c+1-q-b})$, $(\frac{1-q-b}{a-c+1-q-b}, 1)$, $(1, 1)$, $(1, \frac{1-q-b}{a-c+1-q-b})$.

The *equilibrium strategies*:

- Weak incumbent S accommodates with probability one in market 2,
- Weak incumbent T accommodates with probability one in market 2,
- Weak entrant 2 stays out,
- If there was an entry and accommodation by both incumbents in market 2 then weak entrant 1 enters, otherwise it stays out.

The *beliefs* in the beginning of period 1:

- If there was an entry in market 2 and it was followed by aggressive response of incumbent $i = S, T$ then the posterior probability (assigned by incumbent $j \neq i$ and entrant 1) that incumbent i is tough is equal to 1. If incumbent $i = S, T$ accommodated following entry in period 2 then the posterior probability that it is tough is equal to zero.

If there was no entry in period 2 then the posteriors are equal to the priors, i.e. $p_i^1 = p_i^2$ for $i = S, T$.

First, we prove that beliefs in the beginning of period 1 are derived from Bayes' law (one of the requirements of consistency) which is applicable everywhere given priors and equilibrium strategies. Since equilibrium strategies prescribe accommodation for both weak incumbents in market 2, the posterior probability that incumbent $i = S, T$ is tough

given that it fought entry in market 2 should equal to one by Bayes' law. Updating of priors also agrees with Bayes' law when there is no entry in period 2. Hence, the above specified assessment is consistent (a direct proof of consistency is also straightforward for each of the cases in this Appendix). Thus, it is left for us to check sequential rationality of the hypothesized assessment. Weak entrant 2 stays out since prior probabilities (p_3^2, p_7^2) lie outside area V . For the same reason weak entrant 1 stays out if entrant 1 did not enter. And if there was an entry in period 2 and at least one of the incumbents fought then weak entrant 1 strictly prefers to stay out since only tough incumbents fight in market 2. Thus, to complete the proof that the assessment is sequentially rational we have to show that neither of the weak incumbents prefers fighting to accommodation following entry into market 2. We consider this condition for the expected utility of weak incumbent S . The case of weak incumbent T is absolutely symmetric.

Given incumbent T 's equilibrium strategy and the belief structure, the expected utility of weak incumbent S from accommodation after entry in market 2 is equal to:

$$(A1) \quad Eu_S(\text{accommodate}) = p_7^2[-c + Eu_S(\bullet \mid \text{inc. } T \text{ is tough \& inc. } S \text{ accommodated})] \\ + (1 - p_7^2)[0 + Eu_S(\bullet \mid \text{inc. } T \text{ is weak \& inc. } S \text{ accommodated})],$$

where $Eu_S(\bullet \mid \text{inc. } T \text{ is tough \& inc. } S \text{ accommodated})$ is the expected utility of

incumbent S in period 1 given that i) incumbent T is tough (and, hence, fights in period 1) and ii) incumbent S accommodated in period 2; and $Eu_S(\bullet \mid \text{inc. } T \text{ is weak \& inc. } S \text{ accommodated})$ is the expected utility of incumbent S in period 1 given that i) incumbent T is weak (and, hence, accommodates in period 1) and ii) incumbent S accommodated in period 2. These expected utilities have the following form:

$$Eu_S(\bullet \mid \text{inc. } T \text{ is tough \& inc. } S \text{ accommodated}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_S(\bullet \mid \text{inc. } T \text{ is weak \& inc. } S \text{ accommodated}) = 0.$$

The expected utility of weak incumbent S from fighting, given that there was entry in period 2, is given by:

$$(A2) \quad Eu_S(\text{fight}) = p_T^2[-a + Eu_S(\bullet \mid \text{inc. } T \text{ is tough \& inc. } S \text{ fought})] \\ + (1 - p_T^2)[-b + Eu_S(\bullet \mid \text{inc. } T \text{ is weak \& inc. } S \text{ fought})],$$

where $Eu_S(\bullet \mid \text{inc. } T \text{ is tough \& inc. } S \text{ fought})$ is the expected utility of incumbent S in period 1 given that i) incumbent T is tough and ii) incumbent S fought in period 2; and $Eu_S(\bullet \mid \text{inc. } T \text{ is weak \& inc. } S \text{ fought})$ is the expected utility of incumbent S in period 1 given that i) incumbent T is weak and ii) incumbent S fought in period 2. These expected utilities have the following form:

$$Eu_S(\bullet \mid \text{inc. } T \text{ is tough \& inc. } S \text{ fought}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_S(\bullet \mid \text{inc. T is weak \& inc. S fought}) = q \cdot 0 + (1 - q) \cdot 1.$$

Substituting expressions for conditional expected utilities of incumbent S in period 1 into (A1) and (A2), we obtain:

$$Eu_S(\text{accommodate}) = -p_T^2(1 + q) \cdot c + p_T^2(1 - q),$$

$$Eu_S(\text{fight}) = -p_T^2 \cdot a - (1 - p_T^2) \cdot b - p_T^2 q \cdot c + (1 - q).$$

Note that restriction $p_T^2 > \frac{1 - q - b}{a - c + 1 - q - b}$ is equivalent to condition $Eu_S(\text{accommodate}) > Eu_S(\text{fight})$. Hence, both weak incumbents strictly prefer accommodation to fighting if and only if prior probabilities belong to the above specified range. Combining this with the fact that strategies of other players are optimal given the belief structure and the assessment is consistent, we obtain that condition on initial reputations is necessary and sufficient for the specified assessment to be a "sensible" sequential equilibrium.

Equilibrium II:

Conditions on *initial reputations*: (p_S^2, p_T^2) belongs to the shaded area of Figure 2.4.

This shaded area is the intersection of the square with endpoints $(0, 0)$, $(0, \frac{1}{1 + y})$,

$(\frac{1}{1 + y}, \frac{1}{1 + y})$, $(\frac{1}{1 + y}, 0)$ and the complement of area V .

The *equilibrium strategies*:

- Weak incumbent S fights with probability one in market 2,
- Weak incumbent T fights with probability one in market 2,
- Weak entrant 2 stays out,
- If there was an entry in market 2 then weak entrant 1 stays out if both incumbents fought and it enters if at least one of the incumbents accommodated.

If there was no entry in market 2 then weak entrant 1 stays out.

The *beliefs* in the beginning of period 1:

- If there was an entry in market 2 and incumbent $i = S, T$ fought then the posterior probability p_i^1 that incumbent i is tough is equal to the prior p_i^2 . If incumbent $i = S, T$ accommodated in period 2 then the posterior probability that incumbent i is weak is equal to 1.

If there was no entry in period 2 then the posteriors are equal to the priors, i.e. $p_i^1 = p_i^2$ for $i = S, T$.

Since equilibrium strategies prescribe fighting for weak incumbent $i = S, T$ in market 2 and tough incumbents always fight, Bayes' law implies that the posterior is equal to the prior, i.e. $p_i^1 = p_i^2$. Also, the posterior beliefs about an incumbent's type do not depend on the other incumbent's play in period 2. This corresponds to the requirement of consistency

that defections from equilibrium strategies be perceived by players as uncorrelated events (it is straightforward to verify that this requirement is satisfied by all equilibrium assessments in this Appendix). One can easily show that assessment, satisfying this condition and the requirement that posterior beliefs are computed from priors and equilibrium strategies using Bayes' law whenever possible, is consistent in our game. Thus, the assessment is consistent. It is also "sensible" since accommodation is perceived as a sure sign of weakness ($\Pr(\text{inc. } i \text{ is weak} \mid \text{inc. } i \text{ accommodated in market 2}) = 1$).

Now we prove that weak entrants' strategies are sequentially rational. Note that since prior probabilities (p_S^2, p_T^2) lie outside area V it is optimal for weak entrant 2 to stay out. For the same reason weak entrant 1 stays out if there is no entry in period 2. Given incumbents' equilibrium strategies, the condition that weak entrant 1 enters if there was an entry in period 2 and at least one of the incumbents accommodated, while stays out if both incumbents fought is equivalent to the requirement that pair of priors (p_S^2, p_T^2) belongs to the shaded area of Figure 2.4. Thus, to show that the hypothesized assessment is a "sensible" sequential equilibrium we need to prove that given the beliefs and strategies of opponents neither of the weak incumbents prefers accommodation to fighting following entry into market 2. We consider this condition for the expected utility of weak incumbent

S. The case of weak incumbent *T* is absolutely symmetric.

Given incumbent *T*'s equilibrium strategy and the belief structure, the expected utility of weak incumbent *S* from accommodation after entry in market 2 is equal to:

$$(A3) \quad Eu_S(\text{accommodate}) = p_T^2[-c + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated})] \\ + (1 - p_T^2)[-c + Eu_S(\bullet \mid \text{inc.T is weak \& inc.S accommodated})],$$

where conditional utilities of incumbent *S* in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated}) = -c;$$

$$Eu_S(\bullet \mid \text{inc.T is weak \& inc.S accommodated}) = 0.$$

The expected utility of weak incumbent *S* from fighting, given that there was an entry in period 2, is equal to:

$$(A4) \quad Eu_S(\text{fight}) = p_T^2[-a + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought})] \\ + (1 - p_T^2)[-a + Eu_S(\bullet \mid \text{inc.T is weak \& inc.S fought})],$$

where conditional expected utilities in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak \& inc.S fought}) = q \cdot 0 + (1 - q) \cdot 1.$$

Substituting corresponding expressions for period 1 conditional expected utilities into

(A3) and (A4), we obtain:

$$Eu_S(\text{accommodate}) = -(1 + p_1^2)c,$$

$$Eu_S(\text{fight}) = -a - p_1^2qc + (1 - q).$$

It is easy to note that condition $Eu_S(\text{accommodate}) < Eu_S(\text{fight})$ is satisfied under our assumptions on the payoff structure. Thus, the hypothesized assessment is a "sensible" sequential equilibrium for the specified range of prior probabilities.

Equilibrium III:

$$\text{Conditions on initial reputations: } p_1^2 > \frac{1}{1+y}, p_3^2 < \frac{1-q-b}{a-c+1-q-b}.$$

The range of prior probabilities satisfying these conditions is depicted as the shaded area in Figure 2.5. This shaded area is the rectangle with endpoints $(0, \frac{1}{1+y})$, $(0, 1)$,

$$(\frac{1-q-b}{a-c+1-q-b}, 1) \text{ and } (\frac{1-q-b}{a-c+1-q-b}, \frac{1}{1+y}).$$

The equilibrium strategies:

- Weak incumbent S accommodates with probability one in market 2,
- Weak incumbent I' fights with probability one in market 2,
- Weak entrant 2 stays out,
- If there was an entry in period 2 and both incumbents accommodated then weak entrant 1 enters, and it stays out otherwise.

The *beliefs* in the beginning of period 1:

- If there was an entry in period 2 and incumbent S fought then the posterior probability that it is tough is equal to one, and if incumbent S accommodated in period 2 then the probability that it is tough is equal to zero. The posterior belief p_T^1 at information sets following entry in period 2 and fight by incumbent T is equal to the prior p_T^2 . The probability of incumbent T being tough at information sets following entry in period 2 and accommodation by incumbent T is equal to zero.

If there was no entry in period 2 then the posteriors are equal to the priors, i.e. $p_i^1 = p_i^2$ for $i = S, T$.

First, we prove that beliefs in the beginning of period 1 are derived from Bayes' law (whenever applicable) given equilibrium strategies and priors. Since weak incumbent S accommodates and tough incumbent S fights in market 2, by Bayes' law the posterior probability that incumbent S is tough, given that it fought entry in market 2, is equal to one. Since equilibrium strategy prescribes fighting for weak incumbent T in market 2 and tough incumbent T always fights, Bayes' law implies that the posterior is equal to the prior, i.e. $p_T^1 = p_T^2$. Thus, the assessment is consistent. It is also "sensible" since accommodation by incumbent T is perceived as a sure sign of weakness ($\Pr(\text{inc. } T \text{ is weak} \mid \text{inc. } T$

accommodated in market 2) = 1).

Now we prove that weak entrants' strategies are sequentially rational. Note that since prior probabilities (p_S^2, p_T^2) lie outside area V it is optimal for weak entrant 2 to stay out. For the same reason weak entrant 1 stays out if there is no entry in period 2. Weak entrant 1's strategy is also optimal at information sets following entry in period 2. If incumbent T fought in market 2 then $p_T^1 = p_T^2 > \frac{1}{1+y}$ (under our condition on initial reputations), while if incumbent S fought in market 2 then $p_S^1 = 1$, and in both cases weak entrant 1 strictly prefers to stay out. Thus, to show that the hypothesized assessment is "sensible" sequential equilibrium we need to prove that given the beliefs and strategies of other players weak incumbent S prefers accommodation and weak incumbent T prefers fighting in market 2.

Given incumbent T 's equilibrium strategy and the belief structure, the expected utility of weak incumbent S from accommodation after entry in market 2 is equal to:

$$(A5) \quad Eu_S(\text{accommodate}) = p_T^2[-c + Eu_S(\bullet \mid \text{inc. } T \text{ is tough \& inc. } S \text{ accommodated})] \\ + (1 - p_T^2)[-c + Eu_S(\bullet \mid \text{inc. } T \text{ is weak \& inc. } S \text{ accommodated})],$$

where conditional utilities of incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc. } T \text{ is tough \& inc. } S \text{ accommodated}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak \& inc.S accommodated}) = q \cdot 0 + (1 - q) \cdot 1.$$

The expected utility of weak incumbent S from fighting after entry in market 2 is equal to:

$$(A6) \quad Eu_S(\text{fight}) = p_T^2[-a + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought})] \\ + (1 - p_T^2)[-a + Eu_S(\bullet \mid \text{inc.T is weak \& inc.S fought})],$$

where conditional utilities of weak incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak \& inc.S fought}) = q \cdot 0 + (1 - q) \cdot 1.$$

Substituting expressions for conditional utilities of weak incumbent S into (A5) and (A6), we obtain:

$$Eu_S(\text{accommodate}) = -(1 + p_T^2 q)c + (1 - q),$$

$$Eu_S(\text{fight}) = -a - p_T^2 q c + (1 - q).$$

It is easy to see that weak incumbent S strictly prefers accommodation to fighting ($Eu_S(\text{accommodate}) > Eu_S(\text{fight})$) since $a > c$.

Given incumbent S 's equilibrium strategy and the belief structure, the expected utility of weak incumbent T from accommodation after entry in market 2 is equal to:

$$(A7) \quad Eu_T(\text{accommodate}) = p_S^2[-c + Eu_T(\bullet \mid \text{inc.S is tough \& inc.T accommodated})]$$

$$+ (1 - p_S^2)[0 + Eu_T(\bullet \mid \text{inc.S is weak \& inc.T accommodated})],$$

where conditional utilities of incumbent T in period 1 have the following form:

$$Eu_T(\bullet \mid \text{inc.S is tough \& inc.T accommodated}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_T(\bullet \mid \text{inc.S is weak \& inc.T accommodated}) = 0.$$

The expected utility of weak incumbent T from fighting after entry in market 2 is equal to:

$$(A8) \quad Eu_T(\text{fight}) = p_S^2[-a + Eu_T(\bullet \mid \text{inc.S is tough \& inc.T fought})] \\ + (1 - p_S^2)[-b + Eu_T(\bullet \mid \text{inc.S is weak \& inc.T fought})],$$

where conditional expected utilities of incumbent T in period 1 have the following form:

$$Eu_T(\bullet \mid \text{inc.S is tough \& inc.T fought}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_T(\bullet \mid \text{inc.S is weak \& inc.T fought}) = q \cdot 0 + (1 - q) \cdot 1.$$

Substituting expressions for conditional utilities of weak incumbent T into (A7) and

(A8), we obtain:

$$Eu_T(\text{accommodate}) = -p_S^2(1 + q)c + p_S^2(1 - q),$$

$$Eu_T(\text{fight}) = -p_S^2a - (1 - p_S^2)b - p_S^2qc + (1 - q).$$

Note that restriction that $p_S^2 < \frac{1 - q - b}{a - c + 1 - q - b}$ is equivalent to condition

$Eu_T(\text{accommodate}) < Eu_T(\text{fight})$. Thus, we have shown that conditions on initial

reputations are necessary and sufficient for the specified assessment to be a "sensible" sequential equilibrium.

Equilibrium IV: (symmetric to the previous equilibrium).

Description of this equilibrium is exactly the same as the previous one if one switches subscripts S and T .

Equilibrium V:

Conditions on *initial reputations*: i) $p_S^2 \leq \frac{1}{1+y}$,

$$\text{ii) } p_T^2 \leq \frac{1-q-b}{a-c+1-q-b},$$

$$\text{iii) } k \cdot p_S^2 \cdot p_T^2 + l \cdot p_S^2 + m \cdot p_T^2 - m \geq 0,$$

where $k = c(1-q-b+a-c) > 0$, $l = (a-c)(1+y) - (1-q-b)c > 0$,

$m = 1-q-b > 0$.

The range of the prior probabilities that satisfy iii) is the area above curve M of Figure

2.6. Curve M passes through the points $(0, 1)$ and $(\frac{1-q-b}{(a-c)(1+y)-(1-q-b)c}, 0)$.

Rectangle Z has endpoints $(0, 0)$, $(0, \frac{1-q-b}{1-q-b+a-c})$, $(\frac{1}{1+y}, \frac{1-q-b}{1-q-b+a-c})$ and

$(\frac{1}{1+y}, 0)$ The intersection of the area above M with rectangle Z is nonempty for any values of parameters satisfying our initial assumptions (also, curve M is always convex). We assume that $(a-c)((1+y)^2+c) < (1-q-b)y(1+c+y)$. This condition ensures that the intersection of the area above curve M and rectangle with endpoints $(0,0)$, $(0, \frac{1}{1+y})$, $(\frac{1}{1+y}, \frac{1}{1+y})$ and $(\frac{1}{1+y}, 0)$ is empty. Thus, the range of prior probabilities (p_3^2, p_7^2) for which the hypothesized assessment forms a sequential equilibrium is the shaded area of Figure 2.6 (the intersection of the area above M with rectangle Z).

The *equilibrium strategies*:

- Following entry in market 2 weak incumbent S mixes between fighting and accommodation: fights with probability $\beta = \frac{p_3^2 y}{1 - p_3^2}$ and accommodates with complementary probability;
- Weak incumbent T accommodates with probability 1 in market 2;
- Weak entrant 2 stays out;
- If there was an entry in period 2 then weak entrant 1 mixes between entering and staying out after observing that incumbent S fought and incumbent T accommodated: stays out with probability $\gamma = \frac{p_7^2 a + (1 - p_7^2) b - p_7^2 c}{(1 - q)(1 - p_7^2)}$ and enters with complementary probability; weak entrant 1 stays out if incumbent T fought in period 2 and enters if both incumbents

accommodated.

If there was no entry in market 2 then weak entrant 1 stays out.

The *beliefs* in the beginning of period 1:

- If there was an entry in period 2 and incumbent S fought then the posterior probability that it is tough is equal to $p_S^1 = \frac{1}{1+y}$, and if incumbent S accommodated then the probability that it is tough is equal to zero. The posterior belief p_T^1 at information sets following entry in period 2 and fight by incumbent T is equal to 1. The probability of incumbent T being tough at information sets following entry in period 2 and accommodation by incumbent T is equal to zero.

If there was no entry in period 2 then the posteriors are equal to the priors, i.e. $p_i^1 = p_i^2$ for $i = S, T$.

First, we prove that beliefs in the beginning of period 1 are derived from Bayes' law (which is applicable everywhere in this case) given equilibrium strategies and priors. By Bayes' law the posterior probability that incumbent S is tough, given that it fought entry in market 2, is equal to $\Pr(\text{inc. } S \text{ is tough} \mid \text{inc. } S \text{ fought}) = \frac{p_S^2 \cdot 1}{p_S^2 \cdot 1 + (1 - p_S^2) \cdot \beta}$, where β is the probability of weak incumbent S fighting in market 2. Hence, $p_S^1 = \frac{1}{1+y}$ since $\beta =$

$\frac{p_3^2 y}{1 - p_3^2}$. And $\beta \leq 1$ under condition i) on initial reputations. Since weak incumbent T

accommodates and tough incumbent T fights in market 2, by Bayes' law the posterior probability that incumbent T is tough given that it fought entry in market 2 is equal to one.

Thus, the assessment is consistent.

Now we prove that weak entrants' strategies are sequentially rational. Note that since prior probabilities (p_3^2, p_7^2) lie outside area V it is optimal for weak entrant 2 to stay out. For the same reason weak entrant 1 stays out if there is no entry in period 2. Weak entrant 1's strategy is also optimal at information sets following entry in period 2. If incumbent T fought in market 2 then weak entrant 1 strictly prefers to stay out. And weak entrant 1 is willing to mix, after observing a fight by incumbent S and accommodation by incumbent T in market 2, since $p_3^1 = \frac{1}{1+y}$, and, hence, weak entrant 1 is indifferent between entering and staying out. Thus, to show that the hypothesized assessment is a "sensible" sequential equilibrium it is left to prove that given the beliefs and strategies of opponents weak incumbent S is willing to mix and weak incumbent T prefers accommodation in market 2.

Given incumbent T 's and entrant 1's equilibrium strategies and the belief structure, the expected utility of weak incumbent S from accommodation is equal to:

$$(A9) \quad Eu_S(\text{accommodate}) = p_7^2[-c + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated})]$$

$$+ (1 - p_T^2)[0 + Eu_S(\bullet \mid \text{inc.T is weak \& inc.S accommodated})],$$

where conditional utilities of incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak \& inc.S accommodated}) = 0.$$

The expected utility of weak incumbent S from fighting is equal to:

$$(A10) \quad Eu_S(\text{fight}) = p_T^2[-a + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought})] \\ + (1 - p_T^2)[-b + Eu_S(\bullet \mid \text{inc.T is weak \& inc.S fought})],$$

where conditional expected utilities of incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc. S fought}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak \& inc. S fought}) = [q + (1 - q)(1 - \gamma)] \cdot 0 + (1 - q) \cdot \gamma \cdot 1.$$

Substituting expressions for conditional utilities of weak incumbent S into (A9) and

(A10), we obtain:

$$Eu_S(\text{accommodate}) = -p_T^2(1 + q) \cdot c + p_T^2(1 - q),$$

$$Eu_S(\text{fight}) = -p_T^2 \cdot a - (1 - p_T^2) \cdot b - p_T^2 q \cdot c + [p_T^2 + (1 - p_T^2)\gamma](1 - q).$$

Simple algebra shows that if $\gamma = \frac{p_T^2 a + (1 - p_T^2)b - p_T^2 c}{(1 - q)(1 - p_T^2)}$ then

$Eu_S(\text{accommodate}) = Eu_S(\text{fight})$, and, hence, weak incumbent S is willing to mix

between its two actions. Note, that $\gamma \leq 1$ under condition ii) on initial reputations.

Given incumbent S 's and entrant I 's equilibrium strategies and the belief structure, the expected utility of weak incumbent T from accommodation after entry in market 2 is equal to:

$$(A11) \quad Eu_T(\text{accommodate}) = p_S^2[-c + Eu_T(\bullet \mid \text{inc.S is tough \& inc.T accommodated})] \\ + (1 - p_S^2) \{ \beta[-c + Eu_T(\bullet \mid \text{inc.S is weak, inc.S fought \& inc.T accommodated})] \\ + (1 - \beta)[0 + Eu_T(\bullet \mid \text{inc.S is weak, inc.S accommodated \& inc.T accommodated})] \},$$

where conditional utilities of incumbent T in period 1 have the following form:

$$Eu_T(\bullet \mid \text{inc.S is tough \& inc.T accommodated}) = [q + (1 - q)(1 - \gamma)](-c) + (1 - q)\gamma \cdot 1;$$

$$Eu_T(\bullet \mid \text{inc.S is weak, inc.S fought \& inc.T accommodated}) = (1 - q)\gamma;$$

$$Eu_T(\bullet \mid \text{inc.S is weak, inc.S accommodated \& inc.T accommodated}) = 0.$$

The expected utility of weak incumbent T from fighting after entry in market 2 is equal to:

$$(A12) \quad Eu_T(\text{fight}) = p_S^2[-a + Eu_T(\bullet \mid \text{inc.S is tough \& inc.T fought})] \\ + (1 - p_S^2) \{ \beta[-a + Eu_T(\bullet \mid \text{inc.S is weak, inc.S fought \& inc.T fought})] \\ + (1 - \beta)[-b + Eu_T(\bullet \mid \text{inc.S is weak, inc.S accommodated \& inc.T fought})] \},$$

where conditional expected utilities of incumbent T in period 1 have the following form:

$$Eu_T(\bullet \mid \text{inc.S is tough \& inc.T fought}) = q(-c) + (1 - q) \cdot 1;$$

$$Eu_T(\bullet \mid \text{inc.S is weak, inc.S fought \& inc.T fought}) = (1 - q);$$

$$Eu_T(\bullet \mid \text{inc.S is weak, inc.S accommodated \& inc.T fought}) = (1 - q).$$

Substituting expressions for conditional utilities of weak incumbent T and expression for β into (A11) and (A12), we obtain:

$$Eu_T(\text{accommodate}) = -p_S^2[2 + y - \gamma(1 - q)]c + p_S^2\gamma(1 + y)(1 - q),$$

$$Eu_T(\text{fight}) = -p_S^2(1 + y)a - (1 - p_S^2 - p_T^2)yb - p_S^2qc + (1 - q).$$

If condition iii) on prior probabilities (p_S^2, p_T^2) holds then weak incumbent T prefers accommodation to fighting. Thus, we have shown that conditions on initial reputations are necessary and sufficient for the specified assessment to be a "sensible" sequential equilibrium.

Equilibrium VI: (symmetric to the previous equilibrium).

Equilibrium VII:

Conditions on *initial reputations*:

$$\text{i) } \frac{1 - q - b}{(a - c + 1 - q - b)(1 + y)} \leq p_S^2 \leq \frac{1 - q - b}{a - c + 1 - q - b};$$

$$\text{ii) } \frac{1 - q - b}{(a - c + 1 - q - b)(1 + y)} \leq p_T^2 \leq \frac{1 - q - b}{a - c + 1 - q - b}.$$

The range of the prior probabilities that satisfy i) and ii) is the shaded area of Figure

2.7.

The *equilibrium strategies*:

- Following entry in market 2 weak incumbent S mixes between fighting and accommodation:

fights with probability $\beta_S = \frac{1 - q - b - p_S^2(a - c + 1 - q - b)}{(1 - p_S^2)(a - c + 1 - q - b)}$ and accommodates with

complementary probability;

- Following entry in market 2 weak incumbent T mixes between fighting and accommodation:

fights with probability $\beta_T = \frac{1 - q - b - p_T^2(a - c + 1 - q - b)}{(1 - p_T^2)(a - c + 1 - q - b)}$ and accommodates with

complementary probability;

- Weak entrant 2 stays out;
- If there was an entry in period 2 and both incumbents accommodated then weak entrant 1 enters, and it stays out otherwise.

The *beliefs* in the beginning of period 1:

- If there was an entry in period 2 and incumbent S fought then the posterior probability that it is

tough is equal to $p_S^1 = \frac{p_S^2(a - c + 1 - q - b)}{1 - q - b}$, and if incumbent S accommodated in period 2

then the probability that it is tough is equal to zero. If there was an entry in period 2 and

incumbent T fought then the posterior probability that it is tough is equal to

$$p_T^1 = \frac{p_T^2(a-c+1-q-b)}{1-q-b}, \text{ and if incumbent } T \text{ accommodated in period 2 then the}$$

probability that it is tough is equal to zero.

If there was no entry in period 2 then the posteriors are equal to the priors, i.e. $p_i^1 = p_i^2$ for $i = S, T$.

First, we prove that beliefs in the beginning of period 1 are derived from Bayes' law (which is applicable everywhere in this case) given equilibrium strategies and priors. By Bayes' law the posterior probability that incumbent $i = S, T$ is tough, given that it fought

$$\text{entry in market 2, is equal to } \Pr(\text{inc. } i \text{ is tough} \mid \text{inc. } i \text{ fought}) = \frac{p_i^2 \cdot 1}{p_i^2 \cdot 1 + (1 - p_i^2) \cdot \beta_i},$$

where β_i is the probability of weak incumbent i fighting in market 2. Substituting

$$\text{expression for } \beta_i \text{ into this formula yields } p_i^1 = \frac{p_i^2(a-c+1-q-b)}{1-q-b}. \text{ And } 0 \leq \beta_i \text{ for}$$

$$i = S, T \text{ under conditions i) and ii) } p_i^2 \leq \frac{1-q-b}{a-c+1-q-b}. \text{ Thus, the assessment is}$$

consistent.

Now we prove that weak entrants' strategies are sequentially rational. The total probability that incumbent i fights in market 2 is equal to $\Pr(\text{inc. } i \text{ fights in mkt 2})$

$$= p_i^2 \cdot 1 + (1 - p_i^2) \cdot \beta_i = \frac{1-q-b}{a-c+1-q-b} > \frac{1}{1+y}. \text{ Hence, it is optimal for weak}$$

entrant 2 to stay out, $Eu_2(\text{enter}) < Eu_2(\text{stay out})$.

Weak entrant 1 stays out if there is no entry in period 2 since the posteriors, which are equal to the priors in this case, lie outside area V of Figure 2.2. Weak entrant 1's strategy is also optimal at information sets following entry in period 2. Note that $p_i^1 \geq \frac{1}{1+y}$ (for $i = S, T$) under condition $\frac{1-q-b}{(a-c+1-q-b)(1+y)} \leq p_i^2$ on initial reputations. Hence, it is optimal for weak entrant 1 to stay out if there was an entry in period 2 and at least one of the incumbents fought.

Thus, to show that the hypothesized assessment is a "sensible" sequential equilibrium we need to prove that given the beliefs and strategies of opponents weak incumbents are willing to mix in market 2. We consider this condition for the expected utility of weak incumbent S . The case of weak incumbent T is absolutely symmetric.

Given incumbent T 's and entrant 1's equilibrium strategies and the belief structure, the expected utility of weak incumbent S from accommodation is equal to:

$$(A13) \quad Eu_S(\text{accommodate}) = p_T^2[-c + Eu_S(\bullet \mid \text{inc. } T \text{ is tough \& inc. } S \text{ accommodated})] \\ + (1 - p_T^2)\{ \beta_T[-c + Eu_S(\bullet \mid \text{inc. } T \text{ is weak, inc. } T \text{ fought \& inc. } S \text{ accommodated})] \\ + (1 - \beta_T)[0 + Eu_S(\bullet \mid \text{inc. } T \text{ is weak, inc. } T \text{ accommodated \& inc. } S \text{ accommodated})] \},$$

where conditional expected utilities of incumbent T in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated}) = q(-c) + (1 - q) \bullet 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S accommodated}) = 1 - q;$$

$$Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S accommodated}) = 0.$$

The expected utility of incumbent S from fighting is equal to:

$$(A14) \quad Eu_S(\text{fight}) = p_T^2[-a + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought})]$$

$$+(1 - p_T^2)\{ \beta_T[-a + Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S fought})]$$

$$+(1 - \beta_T)[-b + Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S fought})] \},$$

where conditional expected utilities of incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought}) = q(-c) + (1 - q) \bullet 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S fought}) = (1 - q) \bullet 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S fought}) = (1 - q) \bullet 1.$$

Substituting expressions for conditional utilities of weak incumbent S into (A13) and

(A14), we obtain:

$$Eu_S(\text{accommodate}) = -[p_T^2(1 + q) + (1 - p_T^2)\beta_T] \bullet c + [p_T^2 + (1 - p_T^2)\beta_T] \bullet (1 - q),$$

$$Eu_S(\text{fight}) = -[p_T^2 + (1 - p_T^2)\beta_T] \bullet a - [(1 - p_T^2)(1 - \beta_T)] \bullet b - p_T^2 qc + (1 - q).$$

Simple algebra shows that $Eu_i(\text{accommodate}) = Eu_i(\text{fight})$ (for $i = S, T$) given

$$\beta_j = \frac{1 - q - b - p_i^2(a - c + 1 - q - b)}{(1 - p_i^2)(a - c + 1 - q - b)} \text{ where } j \neq i, \text{ and, hence, weak incumbent } i \text{ is}$$

willing to mix between his two actions. Thus, we have shown that conditions on initial reputations are necessary and sufficient for the specified assessment to be a "sensible" sequential equilibrium.

Equilibrium VIII:

Conditions on *initial reputations*: (p_S^2, p_T^2) belongs to the shaded area of Figure 2.8.

This shaded area is the intersection of area V and the area above curve M . Curve M passes through the points $(0, \frac{1}{1+y})$ and $(\frac{b}{b(1+y) + a(a-c)}, 0)$ (the cumbersome mathematical formula, that defines boundaries of this area, is not presented in our discussion because of space considerations).

The equilibrium strategies:

- Following entry in market 2 weak incumbent S mixes between fighting and accommodation:

$$\text{fights with probability } \beta_S = \frac{p_S^2}{1 - p_S^2} \frac{p_T^2 \cdot x + (1 - p_T^2)y}{1 - p_T^2 - p_T^2 \cdot y} \text{ and accommodates with}$$

complementary probability;

- Weak incumbent T fights with probability 1 in market 2;
- Weak entrant 2 stays out;

- If there was an entry in period 2 and both incumbents fought then weak entrant 1 mixes between entering and staying out: stays out with probability $\gamma = \frac{a-c}{(1-q)(1+p_T^2c)}$ and enters with complementary probability; weak entrant 1 enters if at least one of the incumbents accommodated in period 2.

If there was no entry in market 2 then weak entrant 1 enters.

The *beliefs* in the beginning of period 1:

- If there was an entry in period 2 and incumbent S fought then the posterior probability that it is tough is equal to $p_S^1 = \frac{1-p_T^2-p_T^2y}{1+y-p_T^2(2y+1-x)}$, and if incumbent S accommodated in period 2 then the probability that it is tough is equal to zero. The posterior belief p_T^1 at information sets following entry in period 2 and fight by incumbent T is equal to the prior p_T^2 . The probability of incumbent T being tough at information sets following entry in period 2 and accommodation by incumbent T is equal to zero.

If there was no entry in period 2 then the posteriors are equal to the priors, i.e. $p_i^1 = p_i^2$ for $i = S, T$.

First, we prove that beliefs in the beginning of period 1 are derived from Bayes' law (whenever applicable) given equilibrium strategies and priors. By Bayes' law the posterior probability that incumbent S is tough, given that it fought entry in market 2, is equal to

$\Pr(\text{inc. } S \text{ is tough} \mid \text{inc. } i \text{ fought}) = \frac{p_S^2 \cdot 1}{p_S^2 \cdot 1 + (1 - p_T^2) \cdot \beta_S}$, where β_S is the probability of

weak incumbent S fighting in market 2. Substituting expression for β_S into this formula

yields $p_S^1 = \frac{1 - p_T^2 - p_T^2 y}{1 + y - p_T^2(2y + 1 - x)}$. Since equilibrium strategy prescribes fighting for

weak incumbent T in market 2 and tough incumbent T always fights, Bayes' law implies

that the posterior is equal to the prior, i.e. $p_T^1 = p_T^2$. Thus, the assessment is consistent. It is

also "sensible" since accommodation by incumbent T is perceived as a sure sign of

weakness ($\Pr(\text{inc. } T \text{ is weak} \mid \text{inc. } T \text{ accommodated in market 2}) = 1$).

Now we prove that weak entrants' strategies are sequentially rational. Since weak incumbent T fights with probability one in period 2, weak entrant 2 strictly prefers to stay out. If there is no entry in period 2 then it is optimal for weak entrant 1 to enter since the posterior reputations, which are equal to the priors, lie inside area V of Figure 2.2. Weak entrant 1 enters if there was an entry in period 2 and at least one of the incumbents accommodated since $(p_S^1, p_T^1) \in V$ in that case. If both incumbents fought in market 2, then the expected utility of weak entrant 1 in case of entry is equal to

$Eu_1(\text{enter} \mid \text{both incumbents fought in mkt 2})$

$$= p_S^1 p_T^1(-x) + p_S^1(1 - p_T^1)(-y) + p_T^1(1 - p_S^1)(-y) + (1 - p_S^1)(1 - p_T^1) \cdot 1.$$

Substituting expression for p_S^1 into this formula we obtain $Eu_1(\text{enter} \mid \text{both incumbents$

fought in mkt 2) = 0, which is equal to weak entrant 1's utility if it stays out. Hence, weak entrant 1 is indifferent between its two actions at the information set following entry in period 2 and fight by both incumbents.

Thus, to show that the hypothesized assessment is a "sensible" sequential equilibrium we need to prove that given the beliefs and strategies of opponents weak incumbent S is willing to mix in market 2 and weak incumbent T prefers fighting to accommodation.

Given incumbent T 's and entrant 1's equilibrium strategies and the belief structure, the expected utility of weak incumbent S from accommodation is equal to:

$$(A15) \quad Eu_S(\text{accommodate}) = p_T^2[-c + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated})] \\ + (1 - p_T^2)[-c + Eu_S(\bullet \mid \text{inc.T is weak \& inc.S accommodated})],$$

where conditional expected utilities of incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated}) = -c;$$

$$Eu_S(\bullet \mid \text{inc.T is weak \& inc.S accommodated}) = 0.$$

The expected utility of weak incumbent S from accommodation is equal to:

$$(A16) \quad Eu_S(\text{fight}) = p_T^2[-a + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought})] \\ + (1 - p_T^2)[-a + Eu_S(\bullet \mid \text{inc.T is weak \& inc.S fought})],$$

where conditional expected utilities of incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought}) = [q + (1 - q)(1 - \gamma)](-c) + (1 - q)\gamma \cdot 1;$$

$$\begin{aligned} Eu_S(\bullet \mid \text{inc.T is weak \& inc.S fought}) &= [q + (1 - q)(1 - \gamma)] \cdot 0 + (1 - q)\gamma \cdot 1 \\ &= (1 - q)\gamma. \end{aligned}$$

Substituting expressions for conditional utilities of weak incumbent S into (A15) and

(A16), we obtain:

$$Eu_S(\text{accommodate}) = -(1 + p_T^2) \cdot c,$$

$$Eu_S(\text{fight}) = -a - p_T^2[1 - (1 - q)\gamma] \cdot c + (1 - q) \cdot \gamma.$$

It is easy to see that $Eu_S(\text{accommodate}) = Eu_S(\text{fight})$ for $\gamma = \frac{a - c}{(1 - q)(1 + p_T^2 c)}$.

And, hence, weak incumbent is willing to mix between its two actions in period 2.

Given incumbent S 's and entrant I 's equilibrium strategies and the belief structure, the expected utility of weak incumbent T from accommodation is equal to:

$$\begin{aligned} (A17) \quad Eu_T(\text{accommodate}) &= p_S^2[-c + Eu_T(\bullet \mid \text{inc.S is tough \& inc.T accommodated})] \\ &+ (1 - p_S^2)\{ \beta_S[-c + Eu_T(\bullet \mid \text{inc.S is weak, inc.S fought \& inc.T accommodated})] \\ &+ (1 - \beta_S)[0 + Eu_T(\bullet \mid \text{inc.S is weak, inc.S accommodated \& inc.T accommodated})] \}, \end{aligned}$$

where conditional expected utilities of incumbent T in period 1 have the following form:

$$Eu_T(\bullet \mid \text{inc.S is tough \& inc.T accommodated}) = -c;$$

$$Eu_T(\bullet \mid \text{inc.S is weak, inc.S fought \& inc.T accommodated}) = 0;$$

$$Eu_T(\bullet \mid \text{inc.S is weak, inc.S accommodated \& inc.T accommodated}) = 0.$$

The expected utility of incumbent T from fighting is equal to:

$$(A18) \quad Eu_T(\text{fight}) = p_S^2[-a + Eu_T(\bullet \mid \text{inc.S is tough \& inc.T fought})] \\ + (1 - p_S^2)\{ \beta_S[-a + Eu_T(\bullet \mid \text{inc.S is weak, inc.S fought \& inc.T fought})] \\ + (1 - \beta_S)[-b + Eu_T(\bullet \mid \text{inc.S is weak, inc.S accommodated \& inc.T fought})] \},$$

where conditional expected utilities of incumbent T in period 1 have the following form:

$$Eu_T(\bullet \mid \text{inc.S is tough \& inc.T fought}) = [q + (1 - q)(1 - \gamma)](-c) + (1 - q)\gamma \cdot 1;$$

$$Eu_T(\bullet \mid \text{inc.S is weak, inc.S fought \& inc.T fought}) = (1 - q)\gamma \cdot 1;$$

$$Eu_T(\bullet \mid \text{inc.S is weak, inc.S accommodated \& inc.T fought}) = 0.$$

Substituting expressions for conditional utilities of weak incumbent S into (A17) and

(A18), we obtain:

$$Eu_T(\text{accommodate}) = -[2p_S^2 + (1 - p_S^2)\beta_S] \cdot c,$$

$$Eu_T(\text{fight}) = -[p_S^2 + (1 - p_S^2)\beta_S]a - [(1 - p_S^2)(1 - \beta_S)] \cdot b - p_S^2[1 - (1 - q)\gamma]c \\ + [p_S^2 + (1 - p_S^2)\beta_S](1 - q)\gamma.$$

If prior probabilities (p_S^2, p_T^2) belong to the shaded area of Figure 2.8, then weak incumbent T prefers fighting to accommodation. Thus, we have shown that conditions on initial reputations are necessary and sufficient for the specified assessment to be a

"sensible" sequential equilibrium.

Equilibrium IX: (symmetric to the previous equilibrium).

Equilibrium X:

Conditions on *initial reputations*: (p_S^2, p_T^2) belongs to some subset of area V of Figure 2.2 (we do not plot boundaries of this subset since their shape varies with parameters of the model. This subset is nonempty and is strictly smaller than V).

The equilibrium strategies:

- Following entry in market 2 weak incumbent S mixes between fighting and accommodation:
fights with probability β_S and accommodates with complementary probability;
- Following entry in market 2 weak incumbent T mixes between fighting and accommodation:
fights with probability β_T and accommodates with complementary probability;
- Weak entrant 2 enters if and only if

$$(A19) [p_S^2 + (1 - p_S^2)\beta_S][p_T^2 + (1 - p_T^2)\beta_T](-x) + [p_S^2 + (1 - p_S^2)\beta_S][(1 - p_T^2)(1 - \beta_T)](-y) \\ + [(1 - p_S^2)(1 - \beta_S)][p_T^2 + (1 - p_T^2)\beta_T](-y) + (1 - p_S^2)(1 - p_T^2)(1 - \beta_S)(1 - \beta_T) > 0;$$

and stays out if the reverse inequality holds.

- If there was an entry in period 2 and it was fought by both incumbents then weak entrant 1 mixes between its two actions: stays out with probability γ , and enters with complementary probability; and weak entrant 1 enters otherwise;

where β_S , β_T and γ are given by the following three equations:

$$(A20) \{(a - c - b)p_T^2(1 - p_S^2)^2\}\beta_S^2 - \{(1 - p_S^2)[b(p_S^2 - p_T^2) + 2\gamma(a - c - b)p_S^2p_T^2]\}\beta_S \\ - \{p_S^2[x(a - c - b)]p_S^2p_T^2 + \gamma b(p_T^2 - p_S^2)\} = 0;$$

$$(A21) \{(a - c - b)p_S^2(1 - p_T^2)^2\}\beta_T^2 - \{(1 - p_T^2)[b(p_T^2 - p_S^2) + 2\gamma(a - c - b)p_S^2p_T^2]\}\beta_T \\ - \{p_T^2[x(a - c - b)]p_S^2p_T^2 + \gamma b(p_S^2 - p_T^2)\} = 0;$$

$$(A22) \gamma = \frac{[p_S^2 + (1 - p_S^2)\beta_S] \cdot [a - c - b] + b}{p_S^2(1 - q)c}.$$

The *beliefs* in the beginning of period 1:

- If there was an entry in period 2 and incumbent S fought then the posterior probability that it is

tough is equal to $p_S^1 = \frac{p_S^2 \cdot 1}{p_S^2 \cdot 1 + (1 - p_S^2) \cdot \beta_S}$, and if incumbent S accommodated in period 2

then the probability that it is tough is equal to zero. If there was an entry in period 2 and

incumbent T fought then the posterior probability that it is tough is equal to

$p_T^1 = \frac{p_T^2 \cdot 1}{p_T^2 \cdot 1 + (1 - p_T^2) \cdot \beta_T}$, and if incumbent T accommodated in period 2 then the

probability that it is tough is equal to zero.

If there was no entry in period 2 then the posteriors are equal to the priors, i.e. $p_i^1 = p_i^2$ for $i = S, T$.

First, note that posterior probabilities (p_S^1, p_T^1) in the beginning of period 1 are derived from Bayes' law (whenever applicable) given equilibrium strategies and priors. Thus, the assessment is consistent.

Now we prove that weak entrants' strategies are sequentially rational. If weak entrant 2 enters its expected utility, given the opponents' equilibrium strategies and prior beliefs, is the left hand side of inequality (A19), while it nets zero if it stays out. Thus, weak entrant 2's strategy is optimal. If there was no entry in period 2 or if there was an entry and at least one of the incumbents accommodated then the posterior beliefs (p_S^1, p_T^1) lie strictly inside area V of Figure 2.2, and, hence, "enter" is optimal for weak entrant 1. If there was an entry in period 2 that was fought by both incumbents, then the expected utility of weak entrant 1 from "enter" is equal to:

$$\begin{aligned} & \frac{p_S^2}{p_S^2 + (1 - p_S^2)\beta_S} \frac{p_T^2}{p_T^2 + (1 - p_T^2)\beta_T} (-x) + \frac{p_S^2}{p_S^2 + (1 - p_S^2)\beta_S} \frac{(1 - p_T^2)\beta_T}{p_T^2 + (1 - p_T^2)\beta_T} (-y) \\ & + \frac{(1 - p_S^2)\beta_S}{p_S^2 + (1 - p_S^2)\beta_S} \frac{p_T^2}{p_T^2 + (1 - p_T^2)\beta_T} (-y) + \frac{(1 - p_S^2)\beta_S}{p_S^2 + (1 - p_S^2)\beta_S} \frac{(1 - p_T^2)\beta_T}{p_T^2 + (1 - p_T^2)\beta_T}. \end{aligned}$$

One can easily verify that β_S and β_T from equations (A20) and (A21) are such that this expression is equal to zero which is the expected utility of weak entrant 1 if it stays out.

Hence, weak entrant 1 is indifferent between its two actions at the information set following fight by both incumbents in market 2.

Thus, to show that the hypothesized assessment is a "sensible" sequential equilibrium we need to prove that, given the beliefs and strategies of opponents, weak incumbents are willing to mix in market 2. We consider this condition for the expected utility of weak incumbent S . The case of weak incumbent T is absolutely symmetric.

Given incumbent T 's and entrant 1's equilibrium strategies and the belief structure, the expected utility of weak incumbent S from accommodation is equal to:

$$(A23) \quad Eu_S(\text{accommodate}) = p_T^2[-c + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated})] \\ + (1 - p_T^2)\{ \beta_T[-c + Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S accommodated})] \\ + (1 - \beta_T)[0 + Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S accommodated})] \},$$

where conditional expected utilities of incumbent T in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated}) = -c;$$

$$Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S accommodated}) = 0;$$

$$Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S accommodated}) = 0.$$

The expected utility of incumbent S from fighting is equal to:

$$(A24) \quad Eu_S(\text{fight}) = p_T^2[-a + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought})]$$

$$+(1 - p_T^2)\{\beta_T[-a + Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S fought})]$$

$$+(1 - \beta_T)[-b + Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S fought})]\},$$

where conditional expected utilities of incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought}) = [q + (1 - q)(1 - \gamma)](-c) + (1 - q)\gamma \cdot 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S fought}) = (1 - q)\gamma \cdot 1;$$

$$Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S fought}) = 0.$$

Substituting expressions for conditional utilities of weak incumbent S into (A23) and (A24), we obtain:

$$Eu_S(\text{accommodate}) = -[2p_T^2 + (1 - p_T^2)\beta_T] \cdot c,$$

$$Eu_S(\text{fight}) = -[p_T^2 + (1 - p_T^2)\beta_T] \cdot a - [(1 - p_T^2)(1 - \beta_T)] \cdot b - p_T^2[1 - (1 - q)\gamma] \cdot c$$

$$+ [p_T^2 + (1 - p_T^2)\beta_T](1 - q)\gamma.$$

Substituting expressions for β_S , β_T and γ from (A20), (A21) and (A22) one can verify that $Eu_i(\text{accommodate}) = Eu_i(\text{fight})$ for $i = S, T$, and, hence, weak incumbents are willing to mix between fighting and accommodation. Thus, we have shown that the hypothesized assessment is a "sensible" sequential equilibrium.

Equilibrium XI:

Conditions on *initial reputations*: (p_S^2, p_T^2) belongs to the shaded area of Figure 2.9.

The *equilibrium strategies*:

- Following entry in market 2 weak incumbent S mixes between fighting and accommodation:

fights with probability $\beta_S = \frac{p_S^2 y}{1 - p_S^2}$ and accommodates with complementary probability;

- Following entry in market 2 weak incumbent T mixes between fighting and accommodation:

fights with probability $\beta_T = \frac{p_T^2 y}{1 - p_T^2}$ and accommodates with complementary probability;

- Weak entrant 2 enters if (p_S^2, p_T^2) belongs to the shaded area of Figure 2.10, and stays out otherwise;

- If there was an entry in period 2 that was fought by incumbent S and accommodated by T ,

then weak entrant 1 mixes between its two actions: stays out with probability

$$\gamma_1 = \frac{b - p_S^2 b(1 + y) - p_T^2 [A - b(1 + c + y)] - p_S^2 p_T^2 A c}{[1 - (p_S^2 + p_T^2)(1 + y) - p_S^2 p_T^2 (c^2 + 2c(1 + y))](1 - q)},$$

where $A = (1 + y)(1 - q + b - a) + (2 + y - q)c$, and enters with complementary probability;

If there was an entry in period 2 that was fought by incumbent T and accommodated by S ,

then weak entrant 1 mixes between its two actions: stays out with probability

$$\gamma_2 = \frac{b - p_T^2 b(1 + y) - p_S^2 [A - b(1 + c + y)] - p_S^2 p_T^2 A c}{[1 - (p_S^2 + p_T^2)(1 + y) - p_S^2 p_T^2 (c^2 + 2c(1 + y))](1 - q)} \text{ and enters with complementary}$$

probability;

If there was an entry in period 2 and both incumbents accommodated then weak entrant 1 enters, and it stays out if both incumbents fought in period 2.

If there was no entry in period 2 then weak entrant 1 enters.

The *beliefs* in the beginning of period 1:

- If there was an entry in period and incumbent S fought then the posterior probability that it is tough is equal to $p_S^1 = \frac{1}{1+y}$, and if incumbent S accommodated in period 2 then the probability that it is tough is equal to zero. If there was an entry in period 2 and incumbent T fought then the posterior probability that it is tough is equal to $p_T^1 = \frac{1}{1+y}$, and if incumbent T accommodated in period 2 then the probability that he is tough is equal to zero.

If there was no entry in period 2 then the posteriors are equal to the priors, i.e. $p_i^1 = p_i^2$ for $i = S, T$.

First, we prove that beliefs in the beginning of period 1 are derived from Bayes' law (whenever applicable) given equilibrium strategies and priors. By Bayes' law the posterior probability that incumbent $i = S, T$ is tough, given that it fought entry in market 2, is equal

to $\Pr(\text{inc. } i \text{ is tough} \mid \text{inc. } i \text{ fought}) = \frac{p_i^2 \cdot 1}{p_i^2 \cdot 1 + (1 - p_i^2) \cdot \beta_i}$, where β_i is the probability of

weak incumbent i fighting in market 2. Substituting expression for β_i into this formula yields $p_i^1 = \frac{1}{1+y}$. Thus, the assessment is consistent.

Now we prove that weak entrants' strategies are sequentially rational. The total probability that incumbent $i = S, T$ fights in market 2 is equal to $\Pr(\text{inc. } i \text{ fights in mkt 2}) = p_i^2 \cdot 1 + (1 - p_i^2) \cdot \beta_i = p_i^2(1 + y)$. Given these probabilities, condition $Eu_2(\text{enter}) > Eu_2(\text{stay out})$ is equivalent to the requirement that (p_S^2, p_T^2) belongs to the shaded area of Figure 2.10. If there was no entry in period 2 then the posterior beliefs (p_S^1, p_T^1) lie strictly inside area V of Figure 2.2, and, hence, weak entrant 1 enters. Weak entrant 1's strategy is also sequentially rational at information sets following entry in period 2, since $p_i^1 = \frac{1}{1+y}$ (for $i = S, T$) if incumbent i fought and, hence, weak entrant 1 is indifferent between "stay out" and "enter" if one incumbent fought and the other accommodated in period 2, and it strictly prefers to stay out if both incumbents fought.

Thus, to show that the hypothesized assessment is a "sensible" sequential equilibrium we need to prove that given the beliefs and strategies of opponents weak incumbents are willing to mix in market 2. We consider this condition for the expected utility of weak incumbent S . The case of weak incumbent T is absolutely symmetric.

Given incumbent T 's and entrant 1's equilibrium strategies and the belief structure, the

expected utility of weak incumbent S from accommodation is equal to:

$$(A25) \quad Eu_S(\text{accommodate}) = p_T^2[-c + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated})] \\ + (1 - p_T^2)\{ \beta_T[-c + Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S accommodated})] \\ + (1 - \beta_T)[0 + Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S accommodated})] \},$$

where conditional expected utilities of incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S accommodated}) = [q + (1 - q)(1 - \gamma_2)](-c) + (1 - q)\gamma_2 \cdot 1; \\ Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S accommodated}) = (1 - q)\gamma_2; \\ Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S accommodated}) = 0.$$

The expected utility of incumbent S from fighting is equal to:

$$(A26) \quad Eu_S(\text{fight}) = p_T^2[-a + Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought})] \\ + (1 - p_T^2)\{ \beta_T[-a + Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S fought})] \\ + (1 - \beta_T)[-b + Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S fought})] \},$$

where conditional expected utilities of incumbent S in period 1 have the following form:

$$Eu_S(\bullet \mid \text{inc.T is tough \& inc.S fought}) = q(-c) + (1 - q) \cdot 1; \\ Eu_S(\bullet \mid \text{inc.T is weak, inc.T fought \& inc.S fought}) = (1 - q) \cdot 1; \\ Eu_S(\bullet \mid \text{inc.T is weak, inc.T accommodated \& inc.S fought}) = (1 - q)\gamma_1 \cdot 1.$$

Substituting expressions for conditional utilities of weak incumbent S and expression for β_T into (A25) and (A26), we obtain:

$$Eu_S(\text{accommodate}) = -p_T^2[2 + y - \gamma_2(1 - q)] \cdot c + p_T^2(1 + y)(1 - q)\gamma_2,$$

$$Eu_S(\text{fight}) = -p_T^2(1 + y) \cdot a - [1 - p_T^2 - p_T^2y] \cdot b - p_T^2q \cdot c + [p_T^2(1 + y) + (1 - p_T^2 - p_T^2y)\gamma_1](1 -$$

If weak incumbents are willing to randomize then they have to be indifferent between fighting and accommodation. It is easy to verify that $Eu_i(\text{accommodate}) = Eu_i(\text{fight})$ (for $i = S, T$) for γ_1 and γ_2 specified in the description of the equilibrium.

Requirement that prior probabilities (p_S^2, p_T^2) belong to the shaded area of Figure 2.9 is equivalent to conditions $0 \leq \gamma_i \leq 1$ for $i = 1, 2$. Thus, we have shown that conditions on initial reputations are necessary and sufficient for the specified assessment to be a "sensible" sequential equilibrium.

NOTES

1. Fudenberg and Levine (1989) call it a Stackelberg payoff. However, this term is commonly used for $g_1^f(\omega_o)$ in the recent game theory literature.

2. That is, after each period the large player learns a measure of small players in each state that have chosen a particular action. But the large player is unable to observe action chosen by any particular small player since deviations by sets of small players of measure zero do not affect realization of joint distribution over actions and states.

3. For excellent surveys of literature on strategic entry deterrence, see Gilbert (1989), Tirole (1988), Ware (1991) and Wilson (1992).

4. Prices can often be changed within a very short period of time and, hence, do not have a commitment value.

5. Assumption of nondurability of entry deterrence technology requires that investment in this technology by any firm in the sequence of potential entrants affects directly only payoffs to the next entrant.

6. Given an industry structure, firms that are using the deterring technology may or may not be more profitable than those using normal technology.

7. Church and Ware's model is similar to that of Economides' (1993) analysis, where firms make entry decisions simultaneously and afterwards play a Stackelberg game.

8. For a recent treatment of informational asymmetry, see Salonen (1994).

9. This chapter of dissertation is a close version of Melkonian and Johnson (1998).

10. In a game of perfect information, players move sequentially and each player knows all previous moves when making his decision, that is, all information sets are singletons. The backward induction argument is: solve for the optimal choice of the last player depending on each possible history of the game, and then solve for the optimal choice of the next to the last mover given that the last mover will make his/her optimal choice.

11. When $N = 1$, this game coincides with the one depicted in Figure 3.2.

12. For two player games, the sets of coalition-proof and Pareto undominated equilibria coincide.

13. We do not present an algorithm for finding sequential equilibria of the game since it is similar to the one used to find equilibria of multiple versions of the "chain-store" game (Kreps and Wilson (1982b), Milgrom and Roberts (1982b)).

14. Beliefs of player 2 about player 1 are such that any failure to implement the policy announced is perceived as a sure sign of non-commitment.

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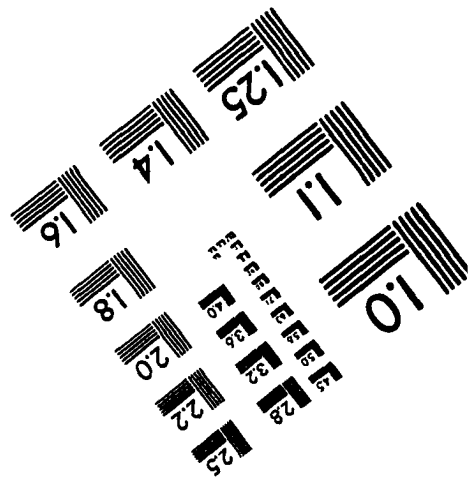
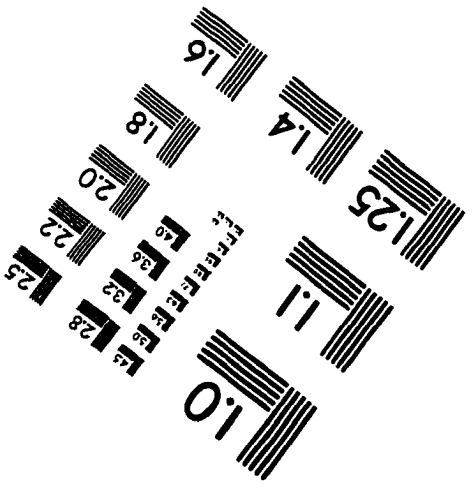
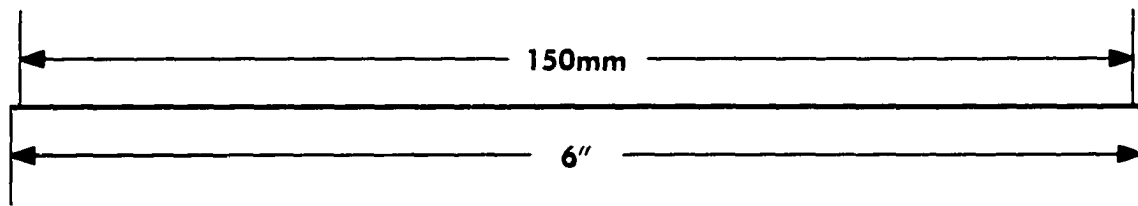
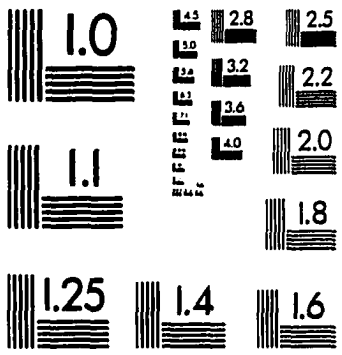
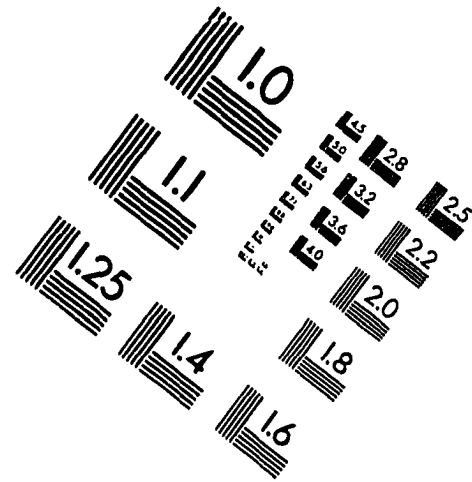
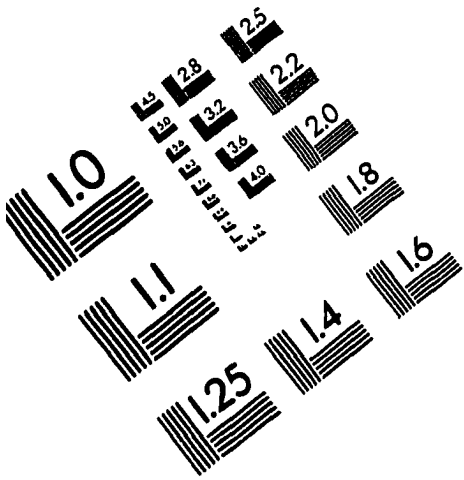
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IMAGE EVALUATION TEST TARGET (QA-3)



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